Fairness, Place/Transition Invariants, and Siphons and Traps

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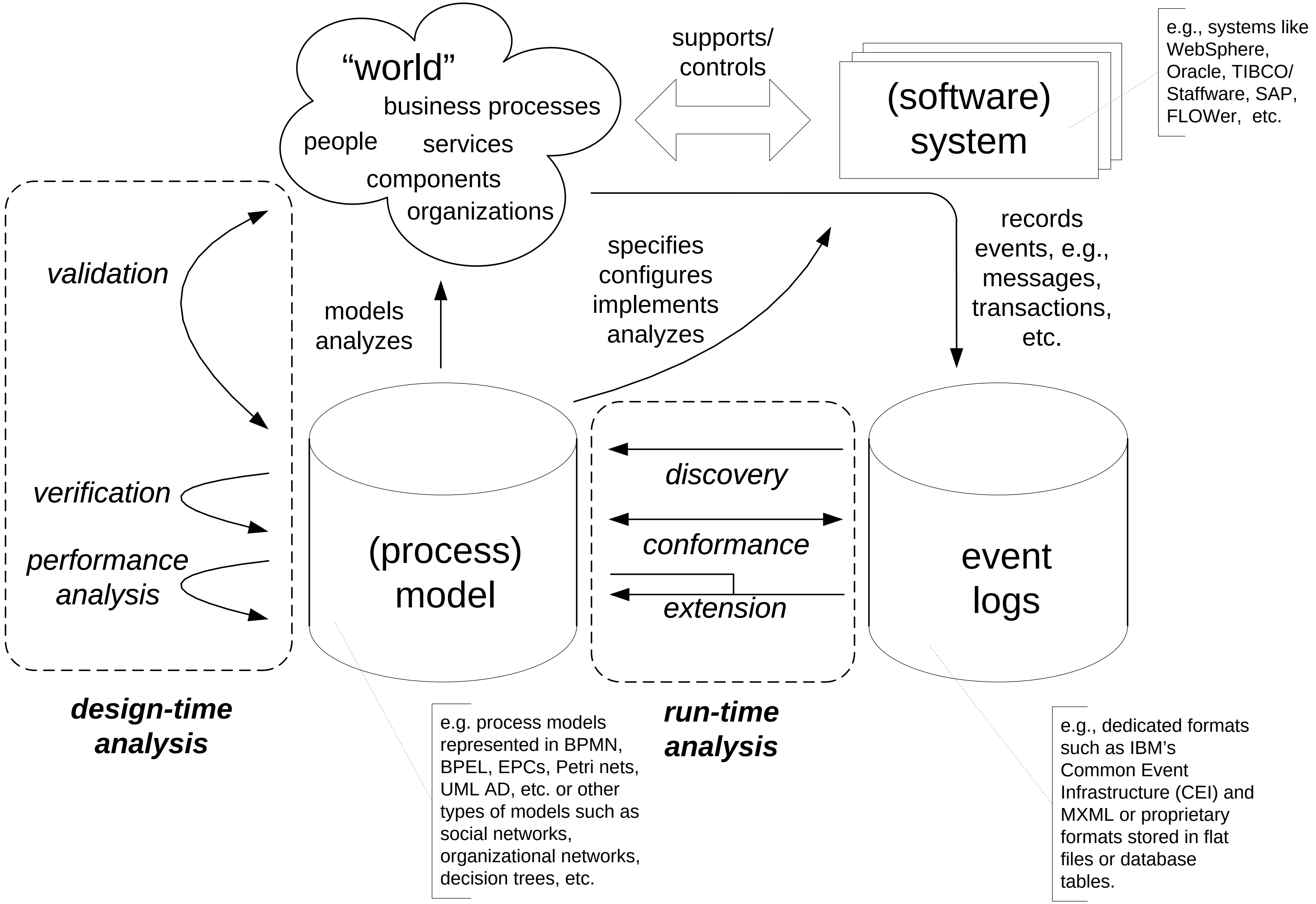
Design-time analysis vs run-time analysis



Outline

- Fairness
- Place/transition invariants
- Siphons and traps

Relevant material

1. **Jörg Desel, Wolfgang Reisig: Place/Transition Petri Nets. Petri Nets 1996: 122-173.** DOI: 10.1007/3-540-65306-6_15
   http://www.springerlink.com/content/x6hn592l35866lu8/fulltext.pdf
2. **Tadao Murata, Petri Nets: Properties, Analysis and Applications, Proceedings of the IEEE. 77(4): 541-580, April, 1989.** http://dx.doi.org/10.1109/5.24143
   http://ieeexplore.ieee.org/iel1/5/911/00024143.pdf
3. **Wil van der Aalst: Process Mining: Discovery, Conformance and Enhancement of Business Processes, Springer Verlag 2011 (chapters 1 & 5)**
   a) **Chapter 1:** DOI: 10.1007/978-3-642-19345-3_1
      http://www.springerlink.com/content/p443h219v3u3537l/fulltext.pdf
   b) **Chapter 5:** DOI: 10.1007/978-3-642-19345-3_5
      http://www.springerlink.com/content/u58h17n3167p0x1u/fulltext.pdf
   c) **Events logs:** http://www.processmining.org/book/

Today's focus is on 1 & 2.

Fairness

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Properties defined earlier ...

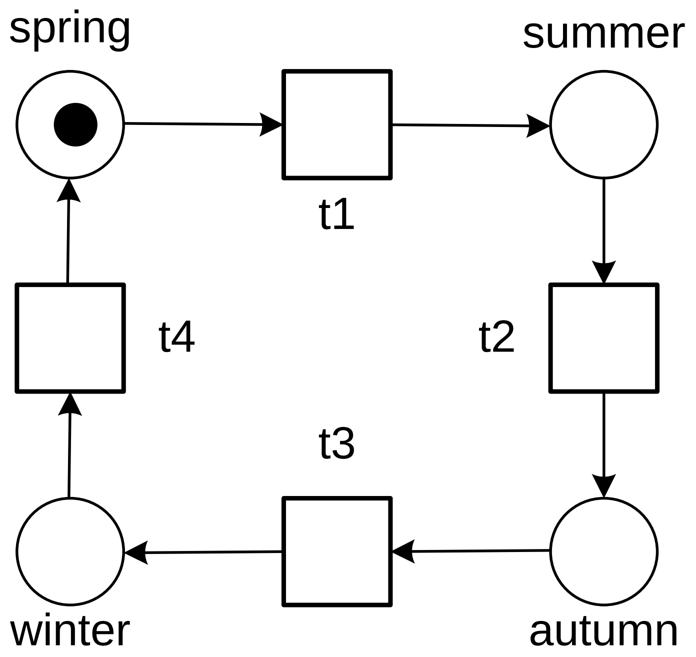
Definition 9 (Basic properties). *Let $N = (P, T, F, W)$ be a Petri net and $M \in \mathbb{B}(P)$ be a marking.*

- (N, M) *is* terminating *if and only if there is a* $k \in \mathbb{N}$ *such that* $|\sigma| \leq k$ *for any firing sequence* σ *(i.e.,* $(N, M)[\sigma\rangle$*).*
- (N, M) *is* deadlock-free *if and only if for any* $M' \in R(N, M)$ *there exists a transition* t *such that* $(N, M')[t\rangle$.
- (N, M) *is* live *if and only if for any* $t \in T$ *and any* $M' \in R(N, M)$ *there exists a* $M'' \in R(N, M')$ *such that* $(N, M'')[t\rangle$.
- (N, M) *is* bounded *if and only if there is a* $k \in \mathbb{N}$ *such that for any* $M' \in R(N, M)$ *and any* $p \in P$*:* $M'(p) \leq k$.
- (N, M) *is* safe *if and only if for any* $M' \in R(N, M)$ *and any* $p \in P$*:* $M'(p) \leq 1$.
- (N, M) *is* reversible *if and only if for any* $M' \in R(N, M)$*:* $M \in R(N, M')$.

We use the fairness notions also used by CPN Tools

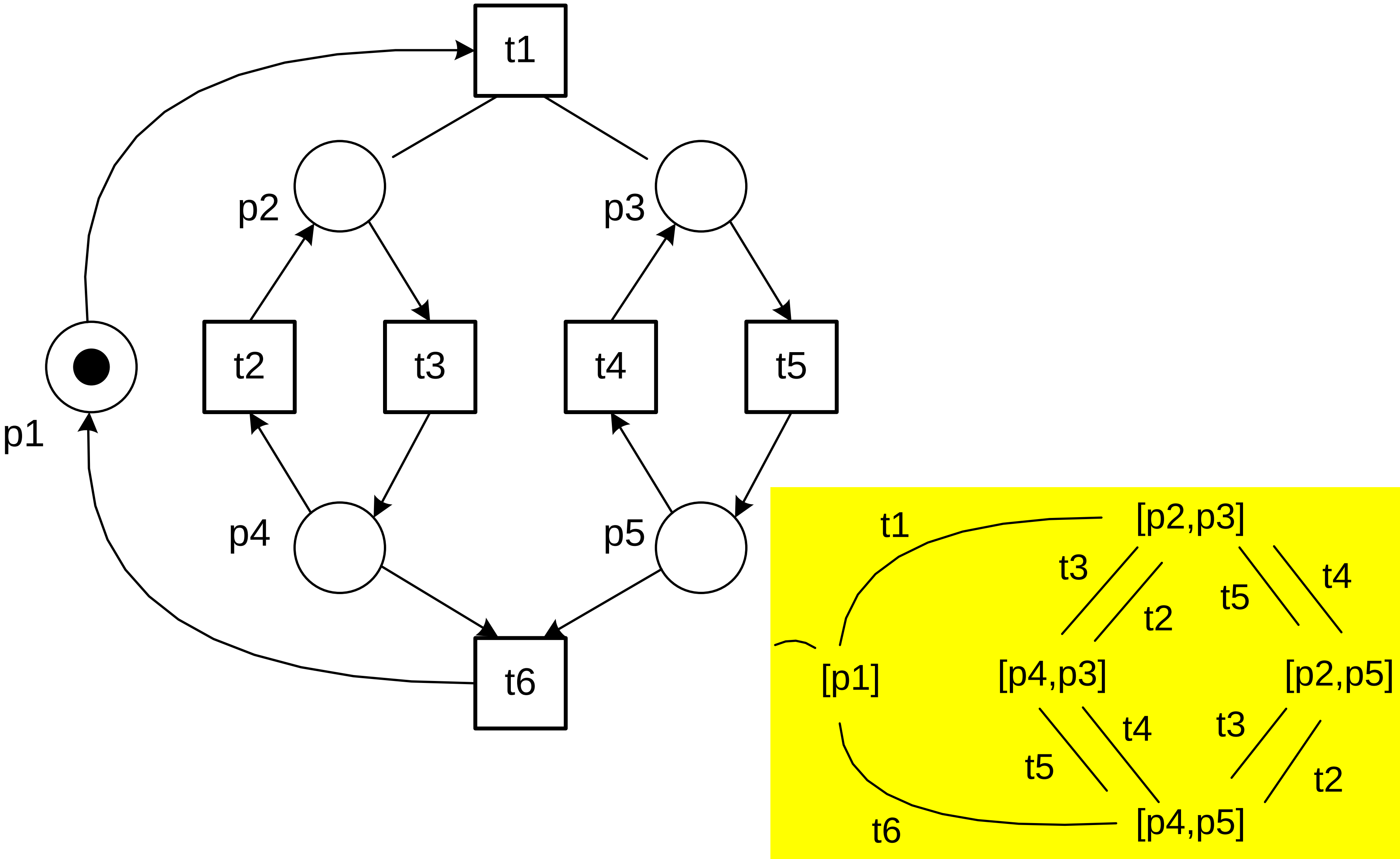
- **Fairness is only relevant if there are** Infinite Firing Sequences **(IFS), otherwise CPN Tools reports: "no infinite occurrence sequences".**
- **Given a transition t it is often desirable that t appears infinitely often in an IFS.**
- **Properties reported by CPN Tools**
  - t is impartial**: t occurs infinitely often in every IFS.**
  - t is fair**: t occurs infinitely often in every IFS where t is enabled infinitely often.**
  - Just**: t occurs infinitely often in every IFS where t is continuously enabled from some point onward**
  - No fairness**: not just, i.e., there is an IFS where t is continuously enabled from some point onward and does not fire anymore.**

Example (1)

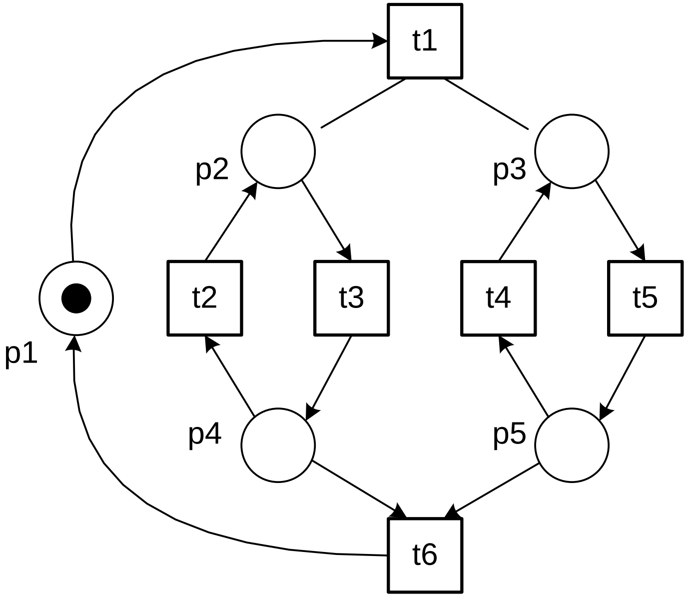


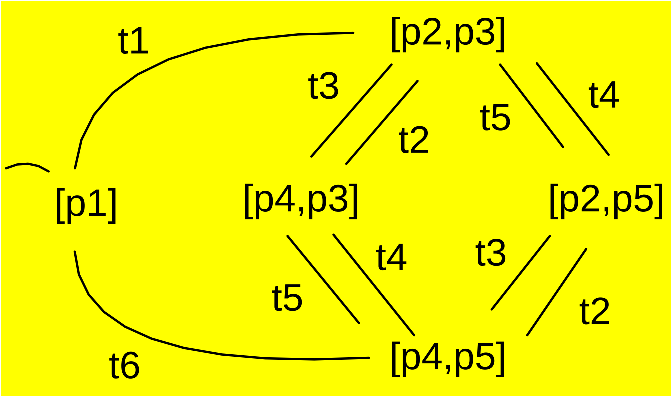
All transitions are **impartial**, i.e., they occur infinitely often in every IFS.

Example (2) Scheibenwischer



Fairness properties





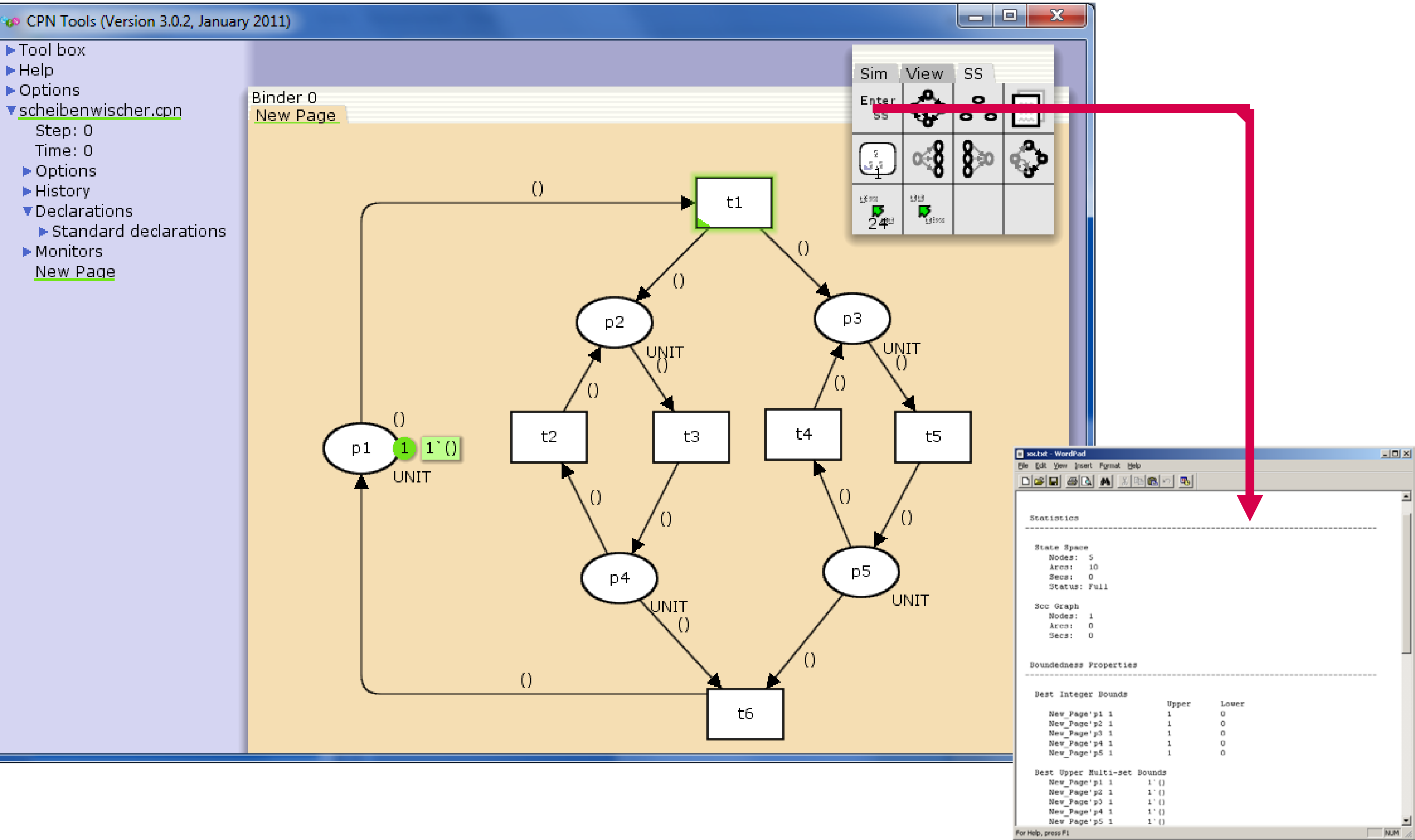
Transition **t1 is fair, i.e.,** t1 occurs infinitely often in every IFS where t1 is enabled infinitely often.

Transition **t2 has no fairness** (i.e., not even just). Even if t2 is enabled continously it does not need to fire.

Transition **t3, t4, and t5 also have no fairness**.

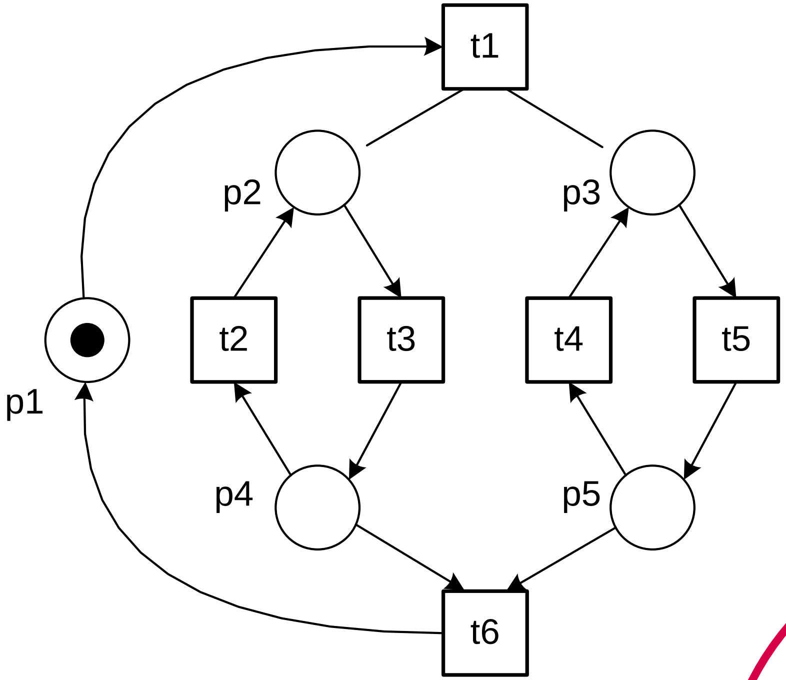
Transition **t6 is just**, i.e., t6 occurs infinitely often in every IFS where t6 is continuously enabled from some point onward.

CPN Tools



CPN Tools (Version 3.0.2, January 2011)
Tool box
Help
Options
scheibenwischer.cpn
Step: 0
Time: 0
Options
History
Declarations
Standard declarations
Monitors
New Page
Binder 0
New Page
Sim
View
SS
Enter SS
t1
t2
t3
t4
t5
t6
p1
p2
p3
p4
p5
UNIT
1`()
Statistics
State Space
Nodes: 5
Arcs: 10
Secs: 0
Status: Full
Scc Graph
Nodes: 1
Arcs: 0
Secs: 0
Boundedness Properties
Best Integer Bounds
Upper Lower
New_Page'p1 1 1 0
New_Page'p2 1 1 0
New_Page'p3 1 1 0
New_Page'p4 1 1 0
New_Page'p5 1 1 0
Best Upper Multi-set Bounds
New_Page'p1 1 1`()
New_Page'p2 1 1`()
New_Page'p3 1 1`()
New_Page'p4 1 1`()
New_Page'p5 1 1`()
For Help, press F1

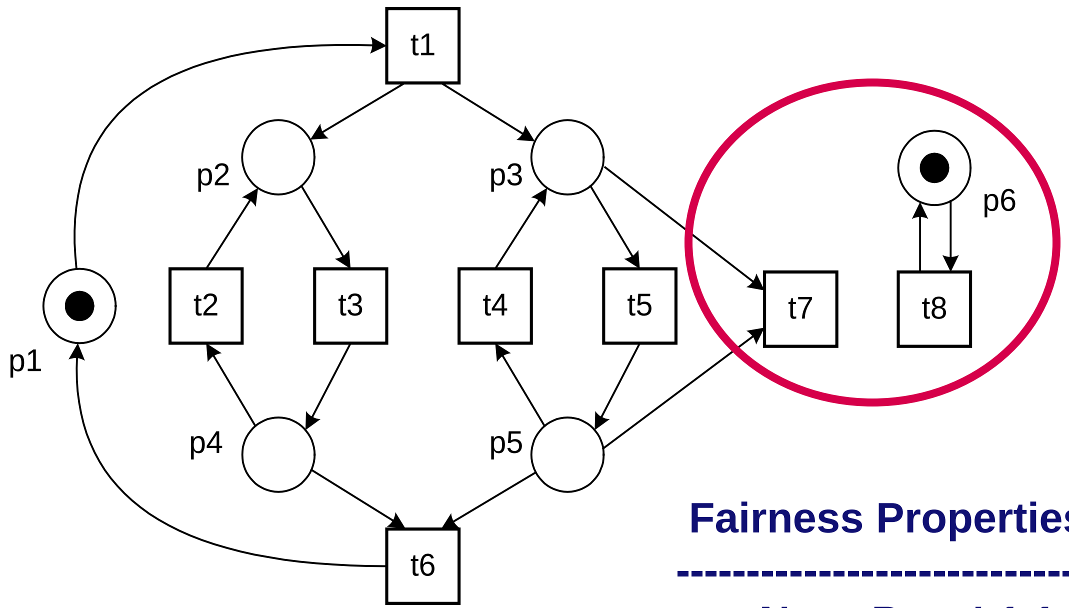
Results



t1	**Fair**
t2	**No Fairness**
t3	**No Fairness**
t4	**No Fairness**
t5	**No Fairness**
t6	**Just**

```
   New_Page'p2 1      1`()
   New_Page'p3 1      1`()
   New_Page'p4 1      1`()
   New_Page'p5 1      1`()

 Best Lower Multi-set Bounds
   New_Page'p1 1      empty
   New_Page'p2 1      empty
   New_Page'p3 1      empty
   New_Page'p4 1      empty
   New_Page'p5 1      empty


Home Properties
------------------------

 Home Markings
   All


Liveness Properties
-----------------------------

 Dead Markings
   None

 Dead Transition Instances
   None

 Live Transition Instances
   All


Fairness Properties
------------------------------------------

 Impartial Transition Instances
   None

 Fair Transition Instances
   New_Page't1 1

 Just Transition Instances
   New_Page't6 1

 Transition Instances with No Fairness
   New_Page't2 1
   New_Page't3 1
   New_Page't4 1
   New_Page't5 1
```

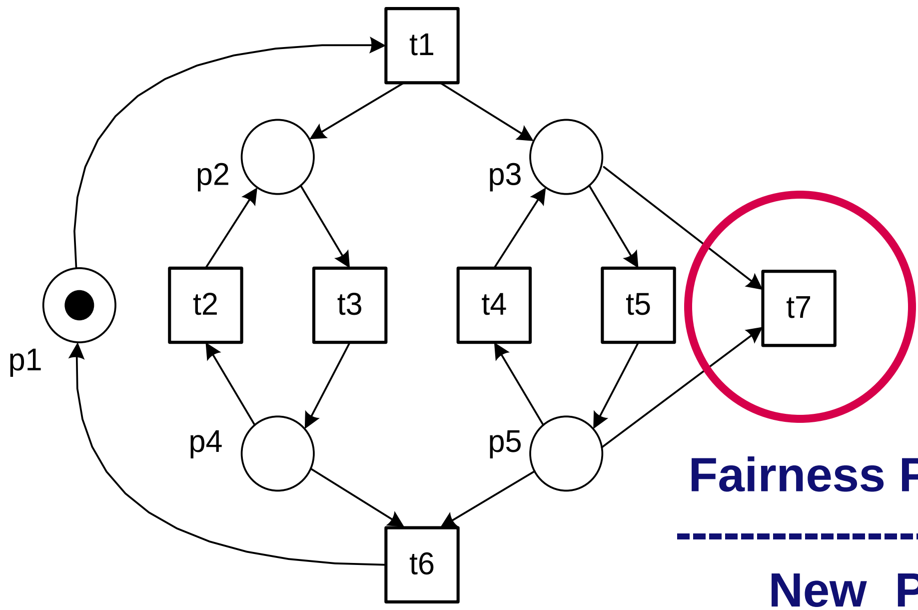
More results



Fairness Properties

New_Page't1 1	No Fairness
New_Page't2 1	No Fairness
New_Page't3 1	No Fairness
New_Page't4 1	No Fairness
New_Page't5 1	No Fairness
New_Page't6 1	No Fairness
New_Page't7 1	Fair
New_Page't8 1	No Fairness

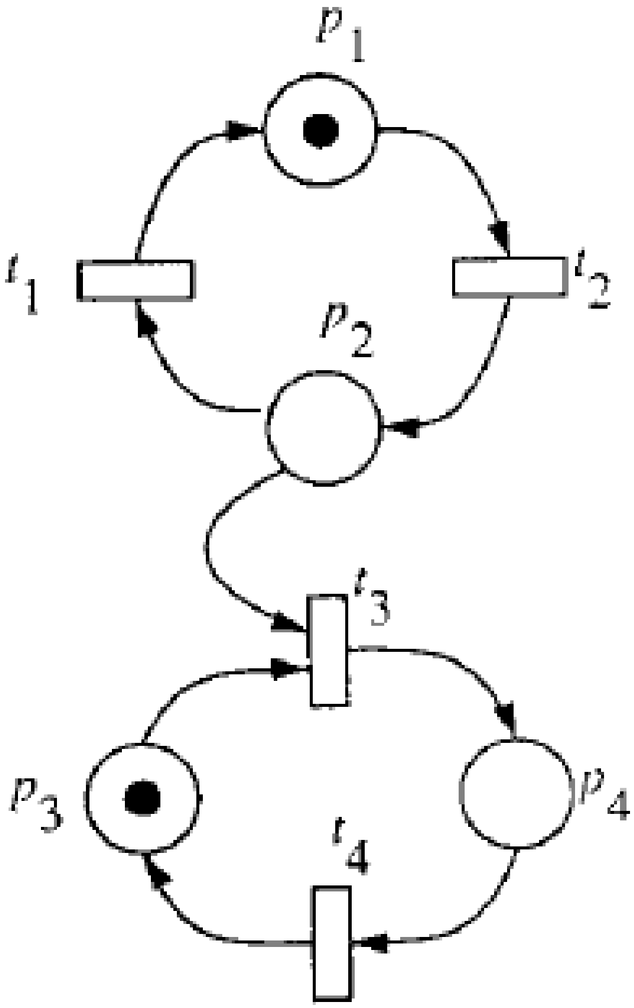
More results

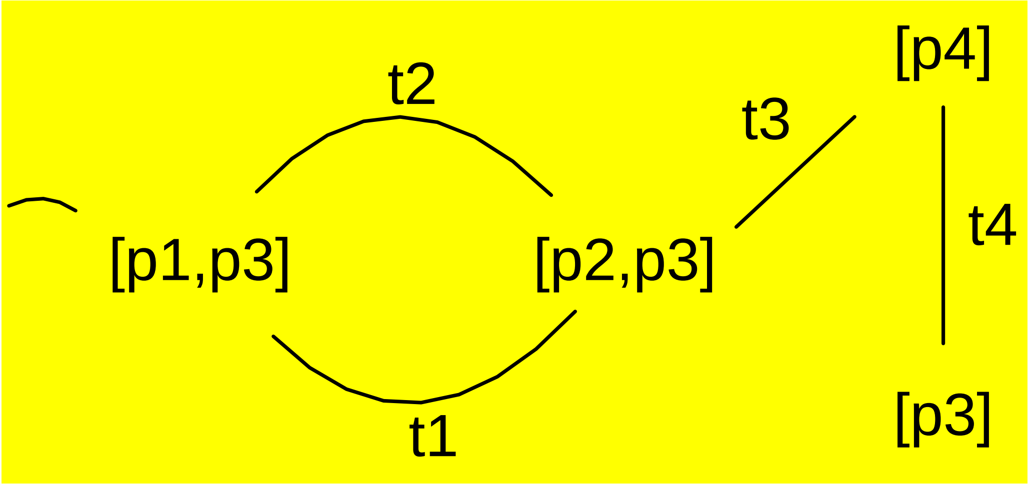


Fairness Properties

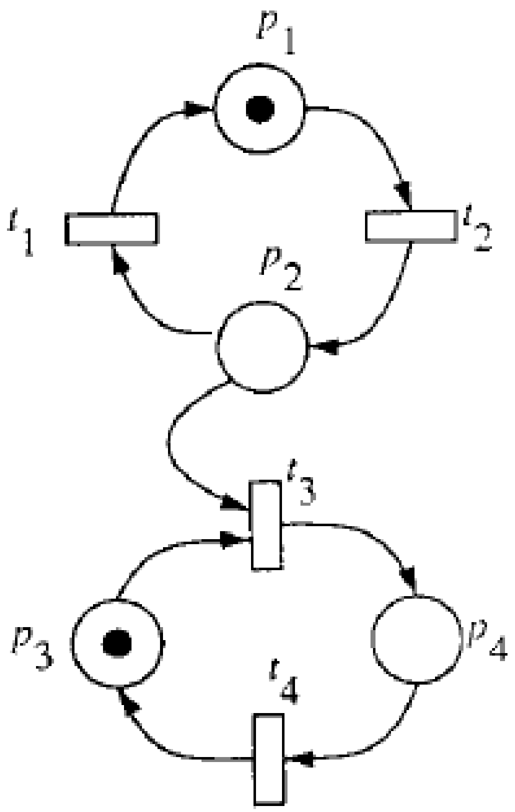
New_Page't1 1	**Fair**
New_Page't2 1	**No Fairness**
New_Page't3 1	**No Fairness**
New_Page't4 1	**No Fairness**
New_Page't5 1	**No Fairness**
New_Page't6 1	**Just**
New_Page't7 1	**Fair**

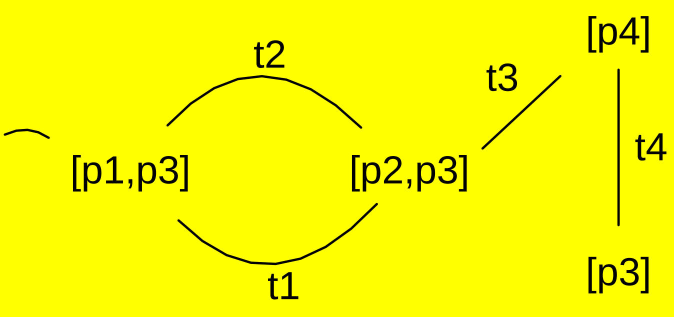
Example (3)





Solution



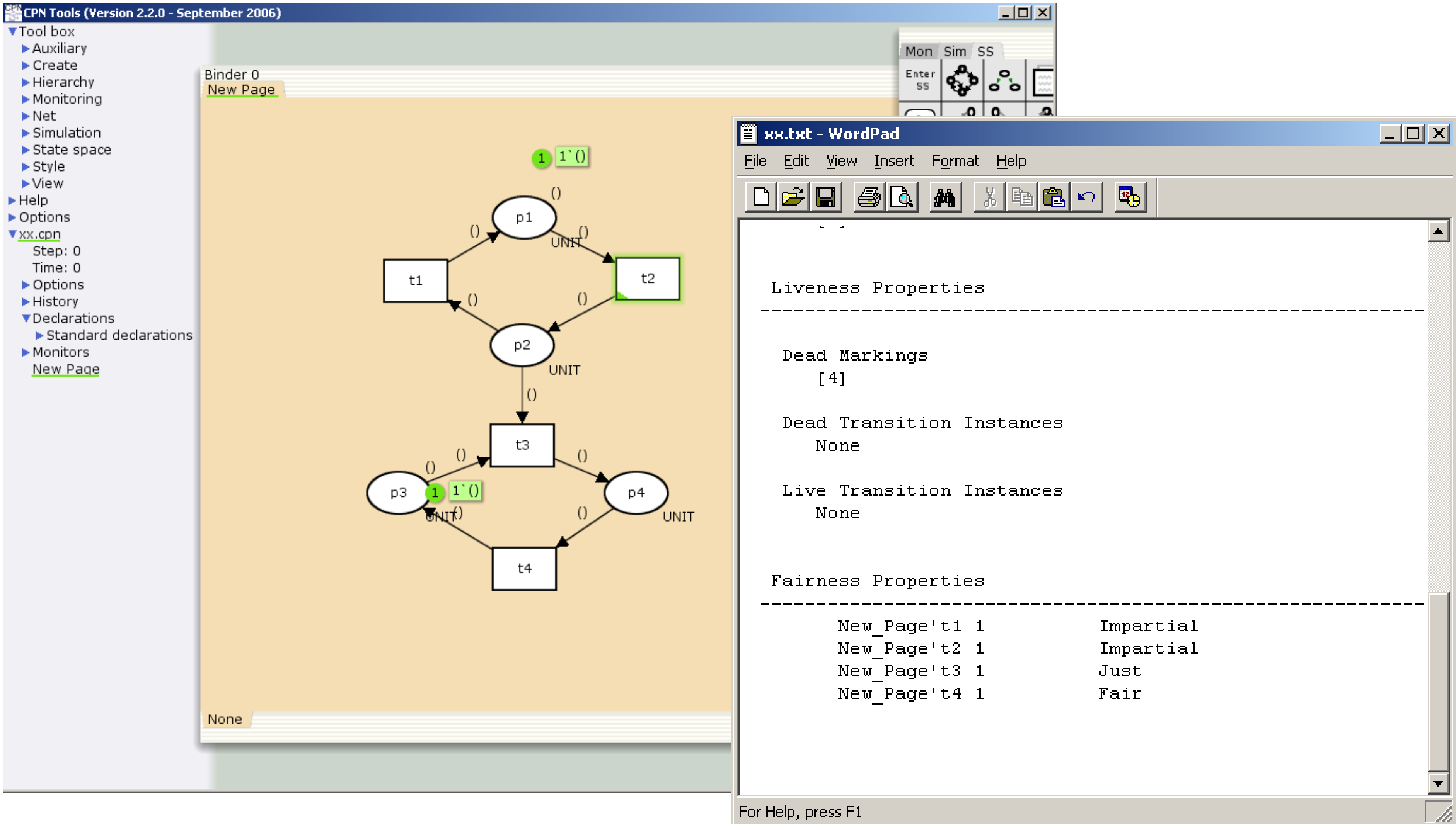


Transitions **t1 and t2 are impartial**, i.e., they occur infinitely often in every IFS (when t3 fires there is no IFS).

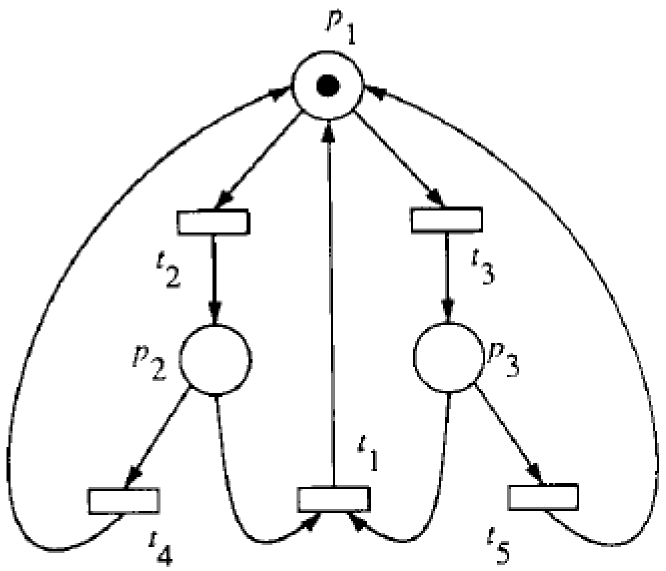
Transition **t3 is just**, i.e., t3 occurs infinitely often in every IFS where t3 is continuously enabled from some point onward. (Note that in the only IFS t3 is enabled infinitely often but does not fire. Hence t3 is not fair. However, t3 is not enabled continuously and thus just.)

Transition **t4 is fair**, i.e., t4 occurs infinitely often in every IFS where t4 is enabled infinitely often. (This never happens.)

Results CPN Tools



Example (4)

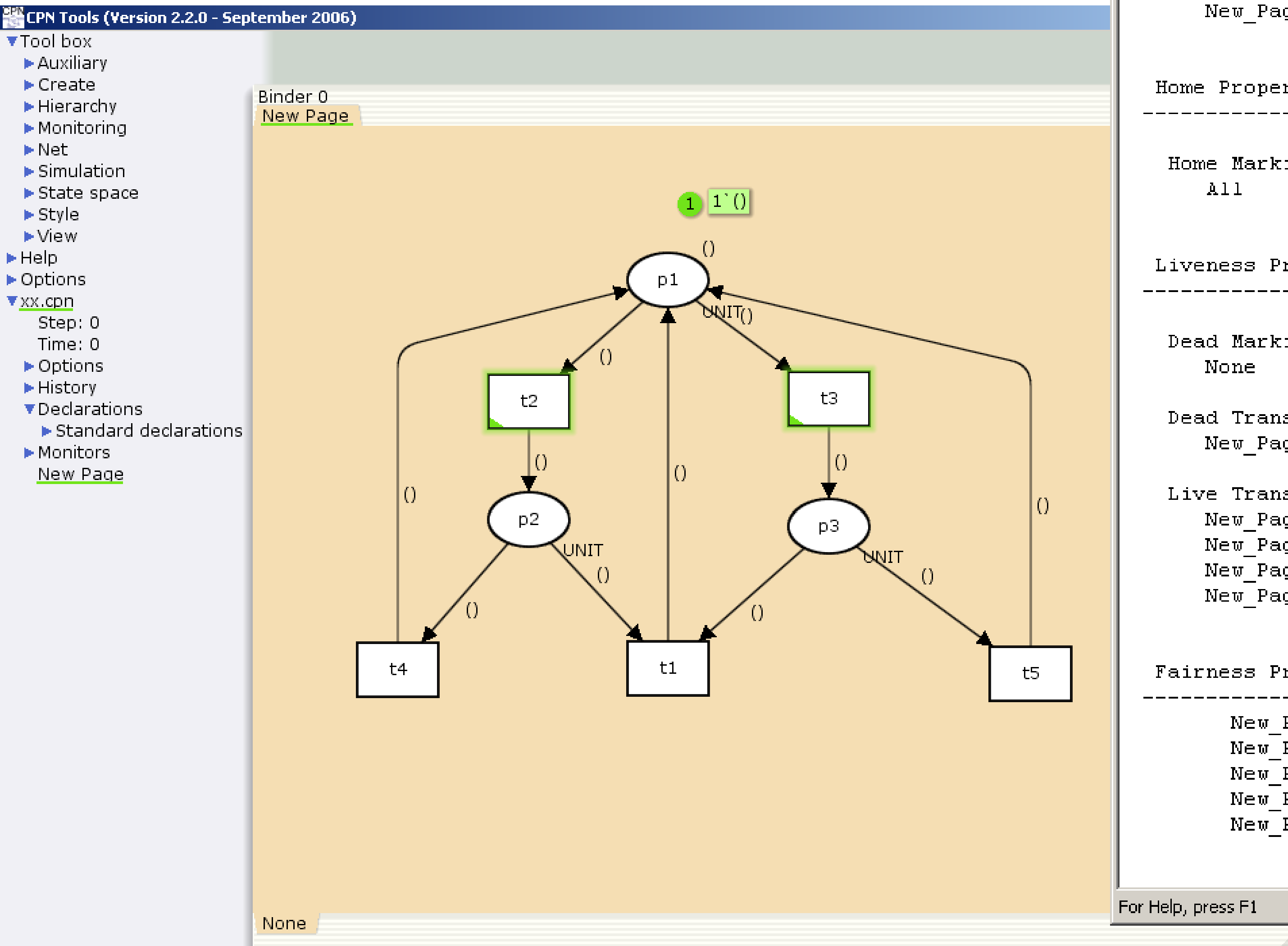


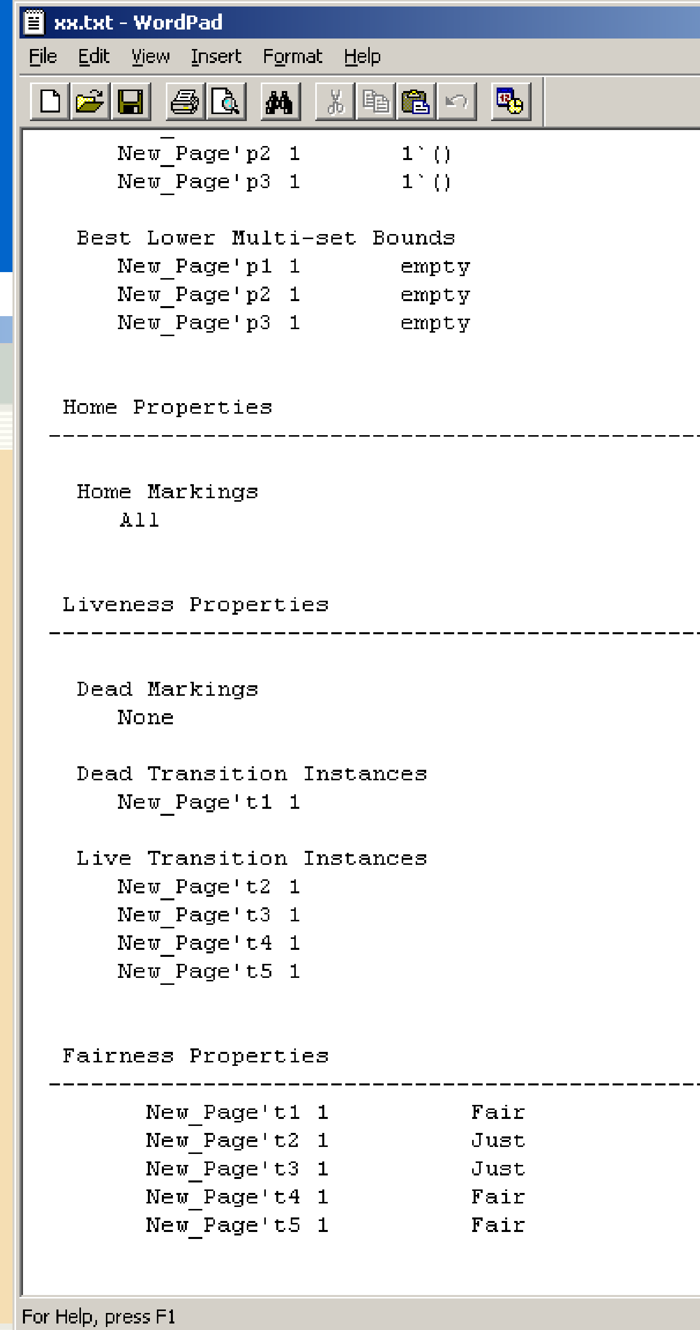
Transitions **t1, t4 and t5 are fair**, i.e., t1/t4/t5 occurs infinitely often in every IFS where t1/t4/t5 is enabled infinitely often. (t1 is never enabled and t4/t5 will fire infinitely often if enabled infinitely often.)

Transitions **t2 and t3 are just**, i.e., t2/t3 occurs infinitely often in every IFS where t2/t3 is continuously enabled from some point onward. (t2/t3 are not enabled continuously and thus just.)

No transition is impartial, i.e., for any transition there is an IFS where it does not occur.

CPN Tools






```
    New_Page'p2 1        1`()
    New_Page'p3 1        1`()

 Best Lower Multi-set Bounds
    New_Page'p1 1        empty
    New_Page'p2 1        empty
    New_Page'p3 1        empty


Home Properties
------------------------------------------

 Home Markings
    All


Liveness Properties
------------------------------------------

 Dead Markings
    None

 Dead Transition Instances
    New_Page't1 1

 Live Transition Instances
    New_Page't2 1
    New_Page't3 1
    New_Page't4 1
    New_Page't5 1


Fairness Properties
------------------------------------------
      New_Page't1 1          Fair
      New_Page't2 1          Just
      New_Page't3 1          Just
      New_Page't4 1          Fair
      New_Page't5 1          Fair
```

Place/Transition Invariants

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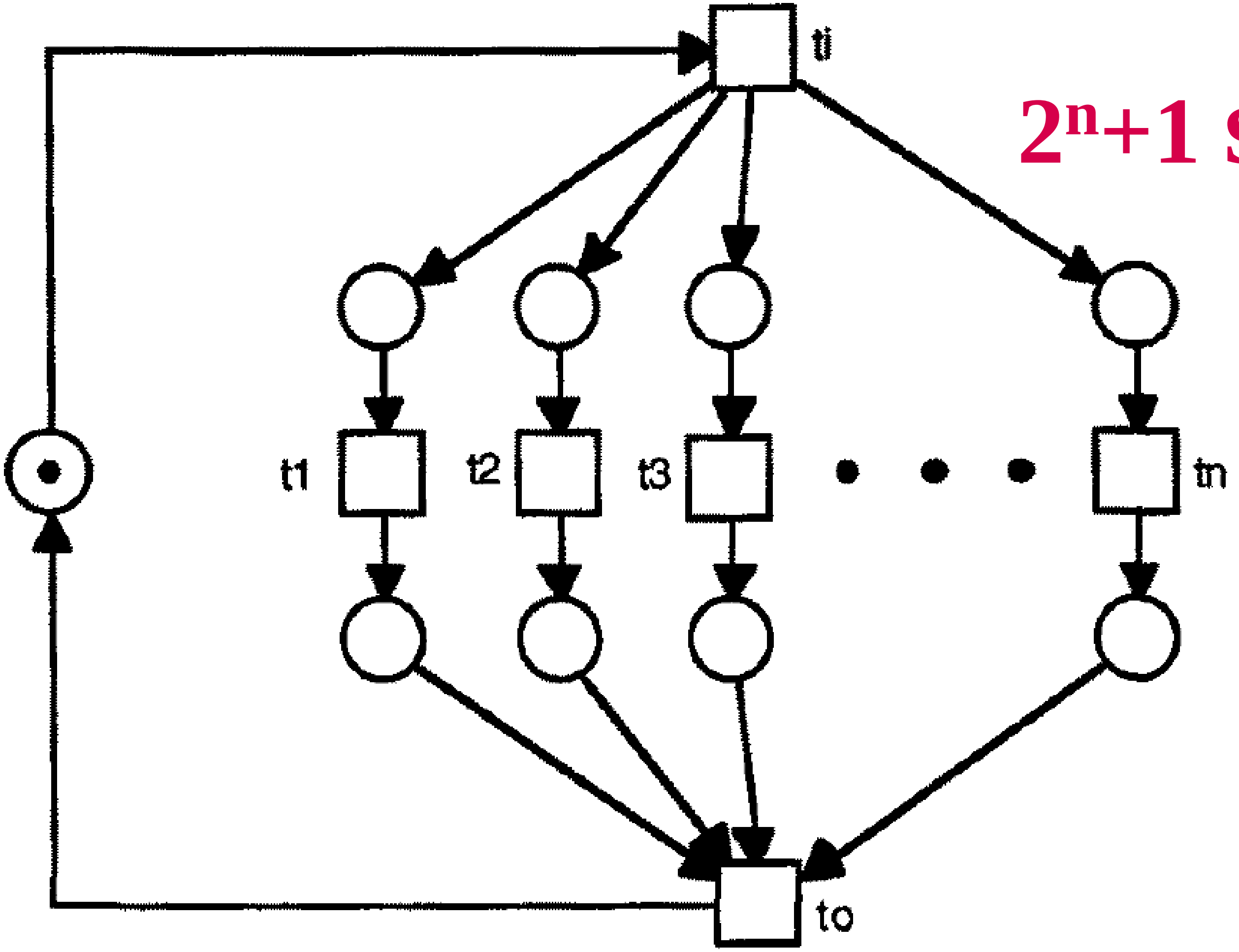
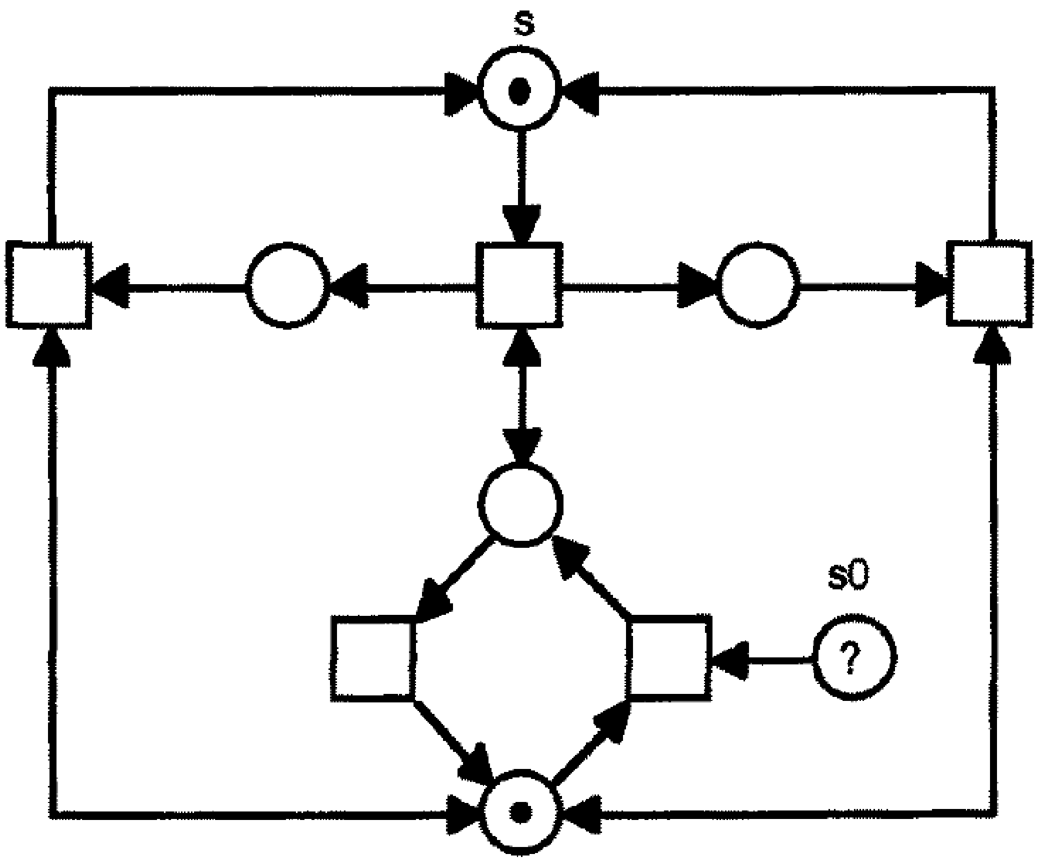
State-explosion problem (1)



2^n+1 states

State-explosion problem (2)



place s is 2^n bounded

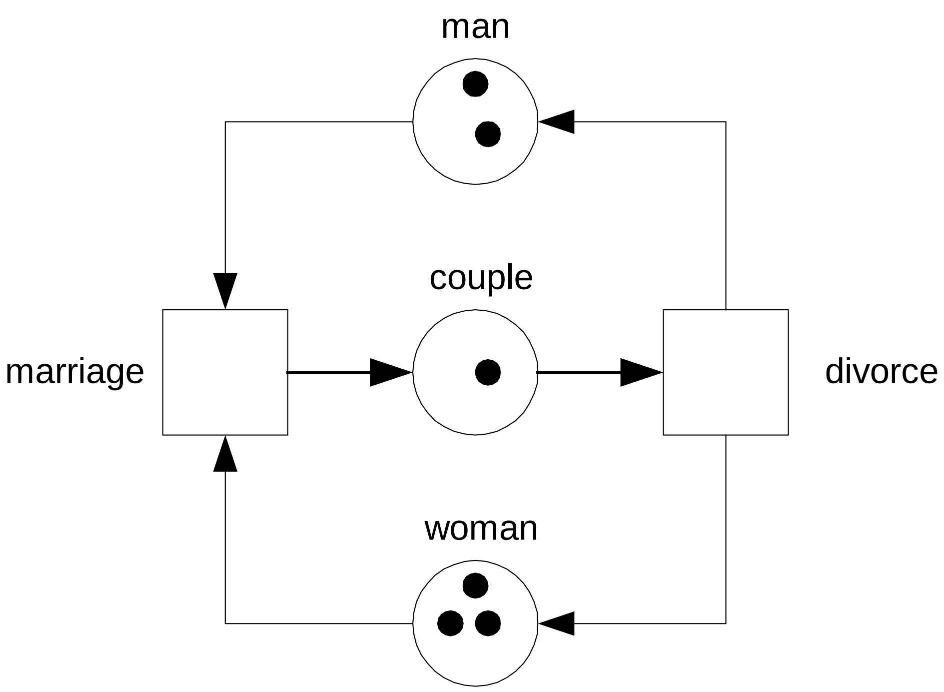
Concepts that can be used to (partially) address the problem ...

- Place invariants
- Transition invariants
- Siphons and traps
- True concurrency semantics
- ...

Invariants

- To avoid state-explosion problem and poor diagnostics.
- Properties independent of initial state.
- Place and transition invariants.
- Invariants can be computed using linear algebraic techniques.

Place invariant

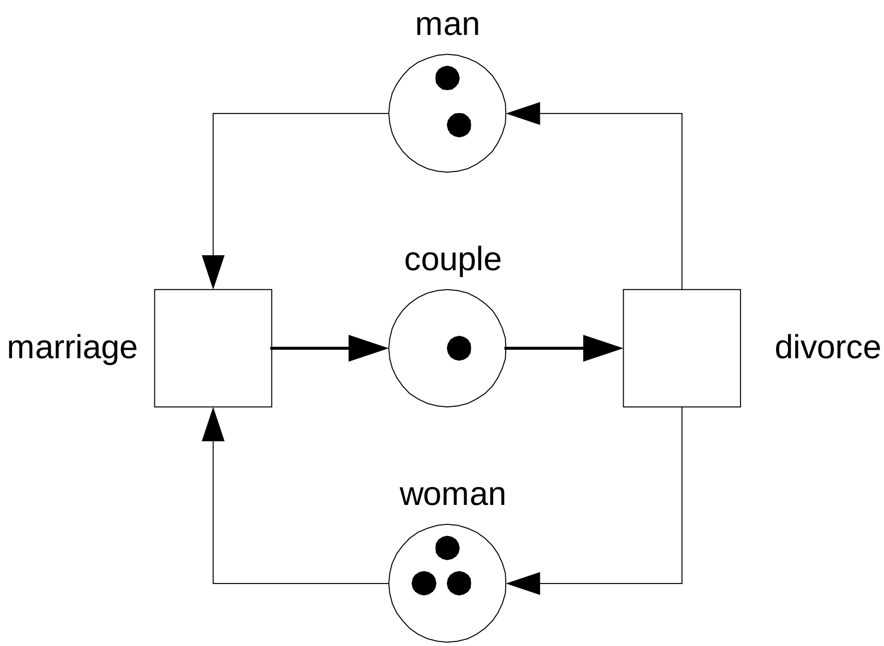


- **Assigns a** weight **to each place.**
- **The weight of a token depends on the weight of the place.**
- **The** weighted token sum **is *invariant*, i.e., no transition can change it**

1 man + 1 woman + 2 couple

can also be denoted as (1,1,2)

Other invariants



- **1 man + 0 woman + 1 couple**

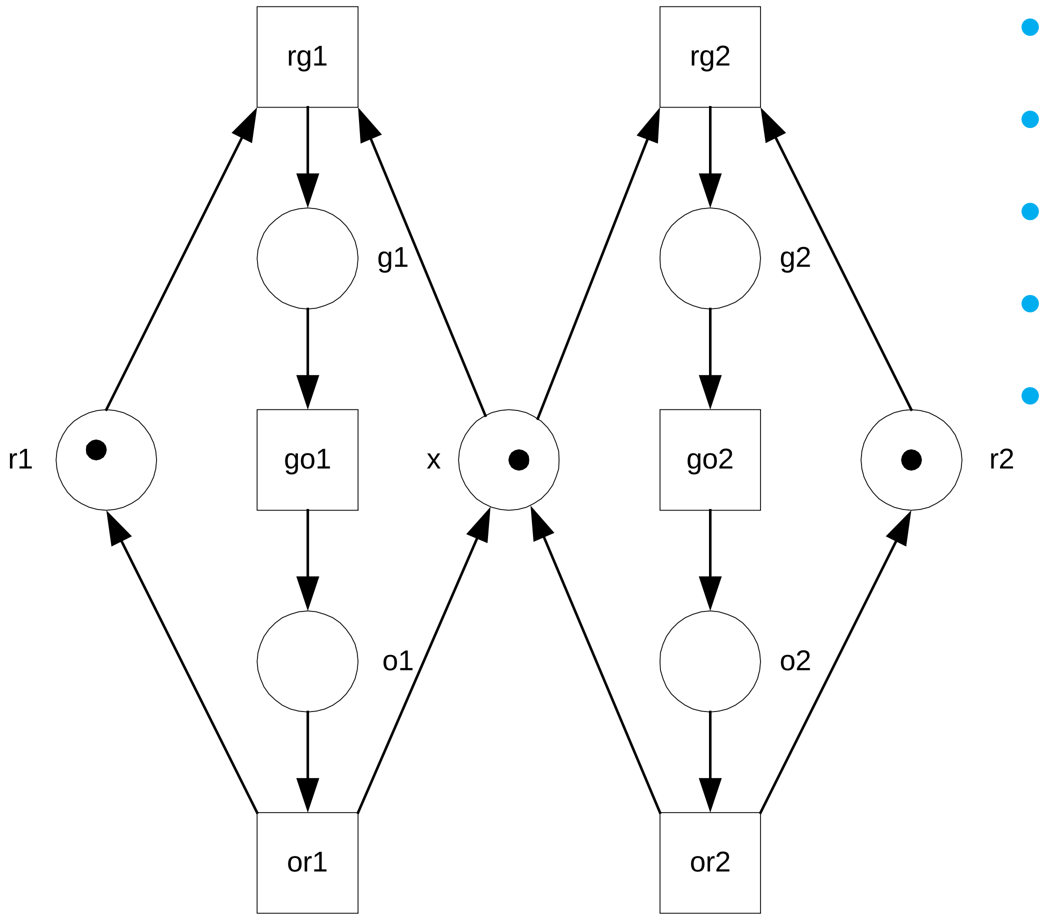
(Also denoted as: man + couple)

- **2 man + 3 woman + 5 couple**
- **-2 man + 3 woman + couple**
- **man – woman**
- **woman – man**

(Any linear combination of invariants is an invariant.)

man-woman can also be denoted as (1,-1,0)

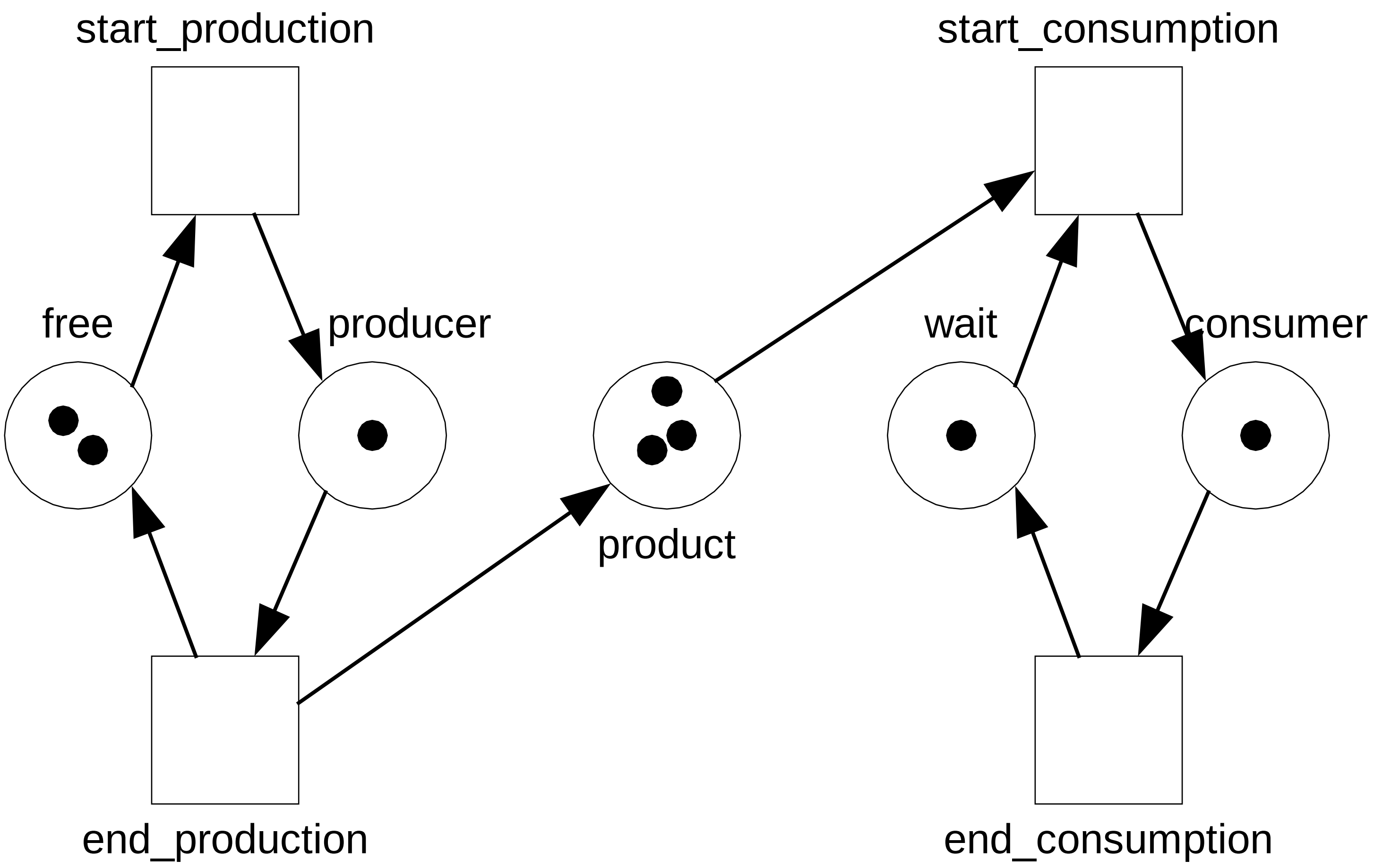
Example: traffic light



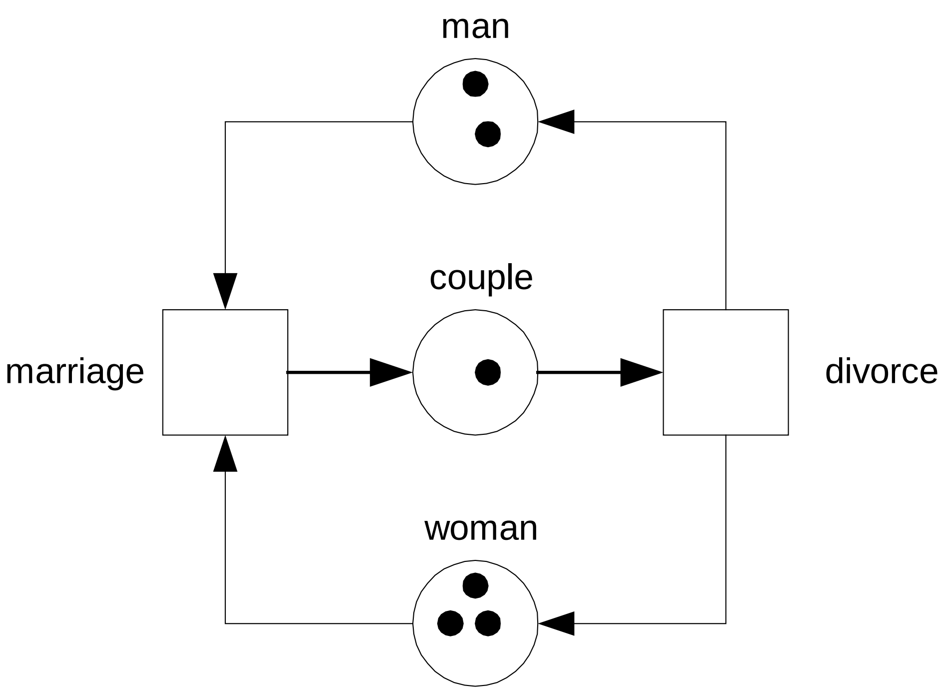
- r1 + g1 + o1
- r2 + g2 + o2
- r1 + r2 + g1 + g2 + o1 + o2
- x + g1 + o1 + g2 + o2
- r1 + r2 - x

r1+r2-x can also be denoted as (1,0,0,1,0,0,-1)

Exercise: Give place invariants



Transition invariant

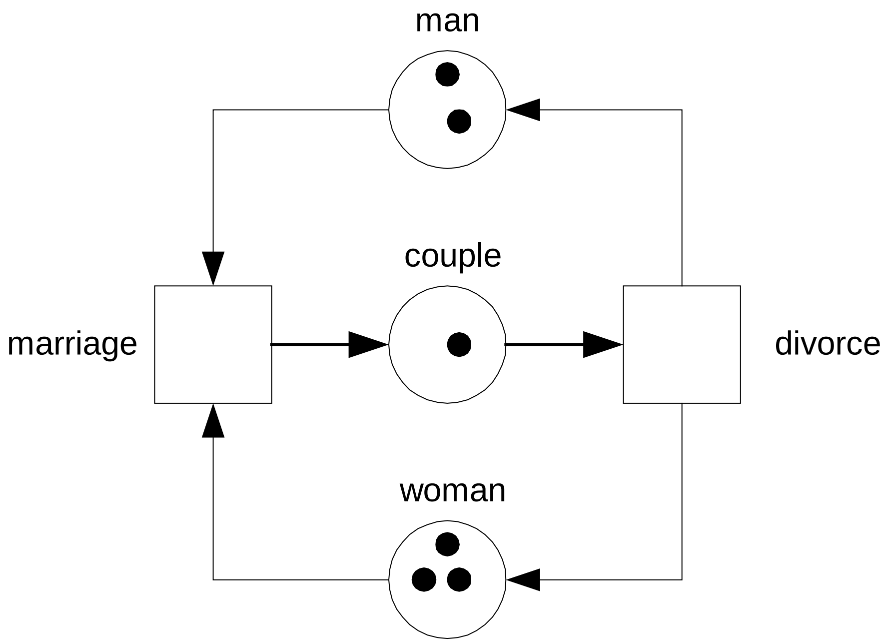


2 marriage + 2 divorce

- **Assigns a** weight **to each** transition**.**
- **If each transition fires the number of times indicated, the system is** back in the initial state**.**
- **I.e. transition invariants indicate *potential* firing sets without any net effect.**

can also be denoted as (2,2)

Other invariants



- 1 marriage + 1 divorce

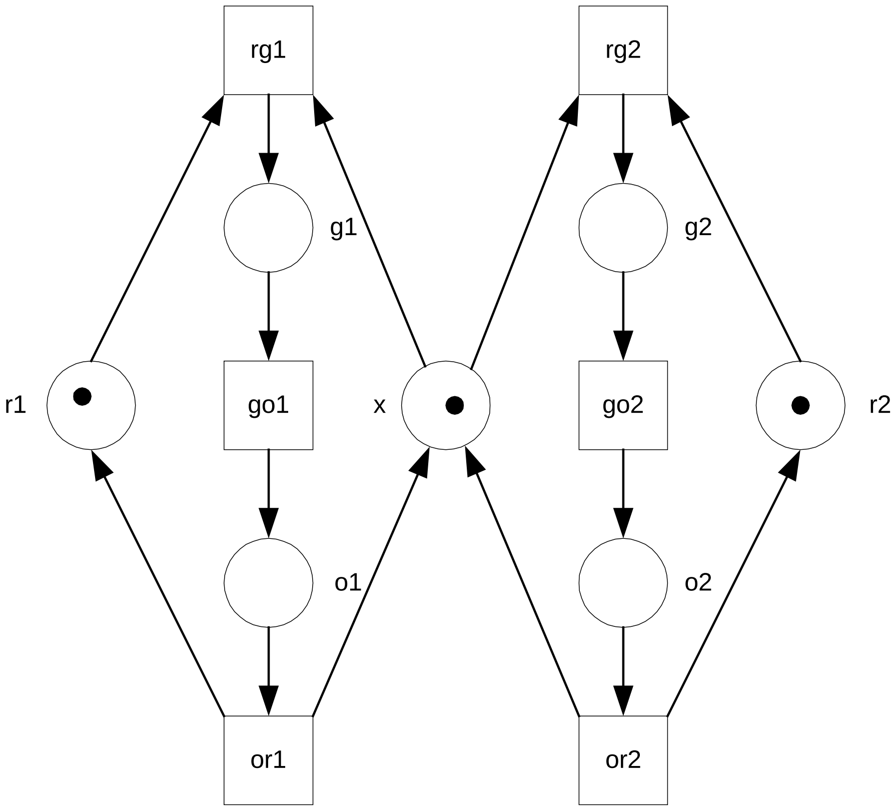
(Also denoted as: marriage + divorce)

- 20 marriage + 20 divorce

Any linear combination of invariants is an invariant.

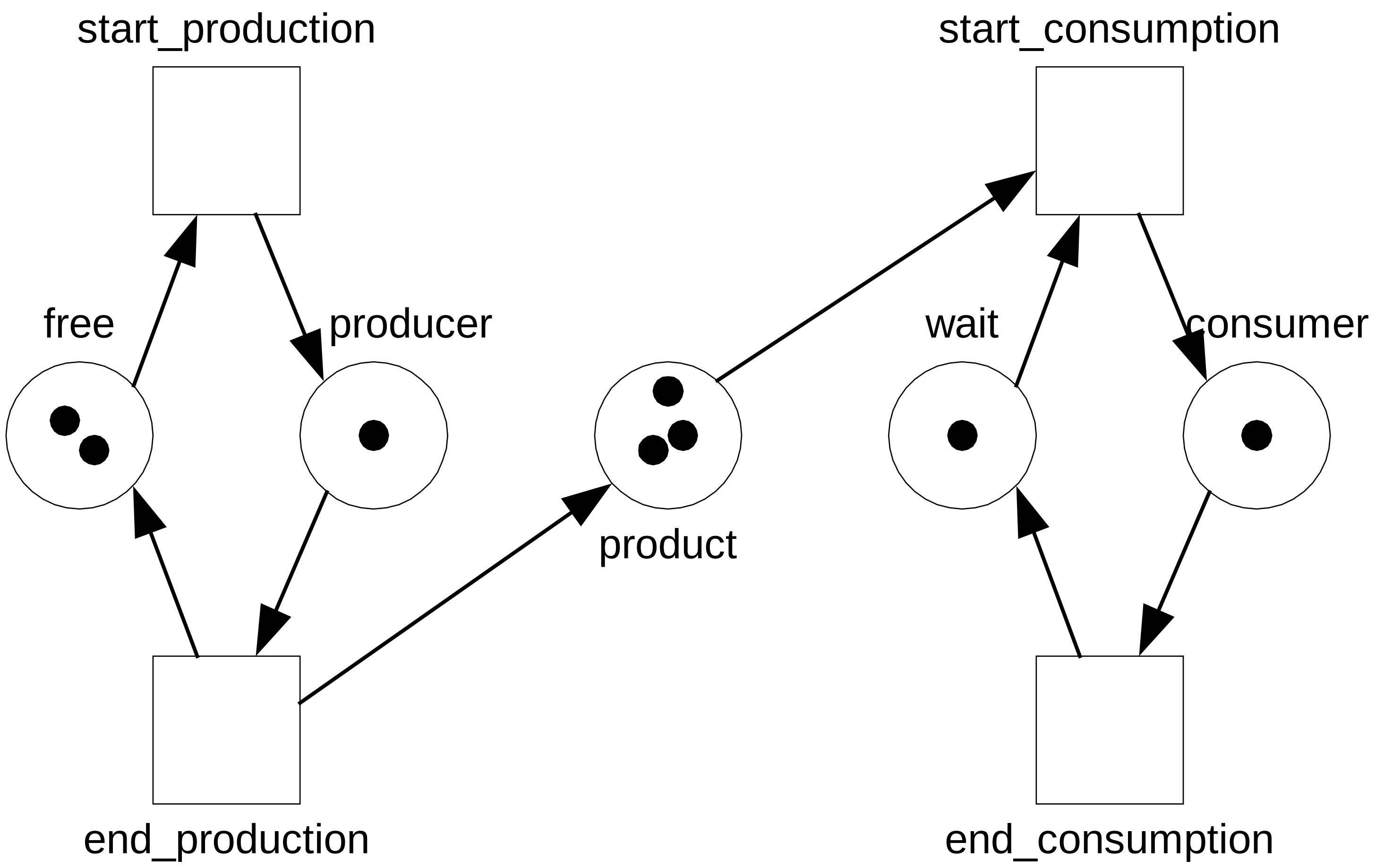
- Invariants may be not be *realizable*.
- There is not a simple interpretation for invariants with negative weights:
  - ❑ Backward firing
  - ❑ t1-t2 (the effects coincide)
  - ❑ t1+2t2+t3-2t4-t5-t6 (the effect of t1+2t2+t3 equals 2t4+t5+t6)

Example: traffic light

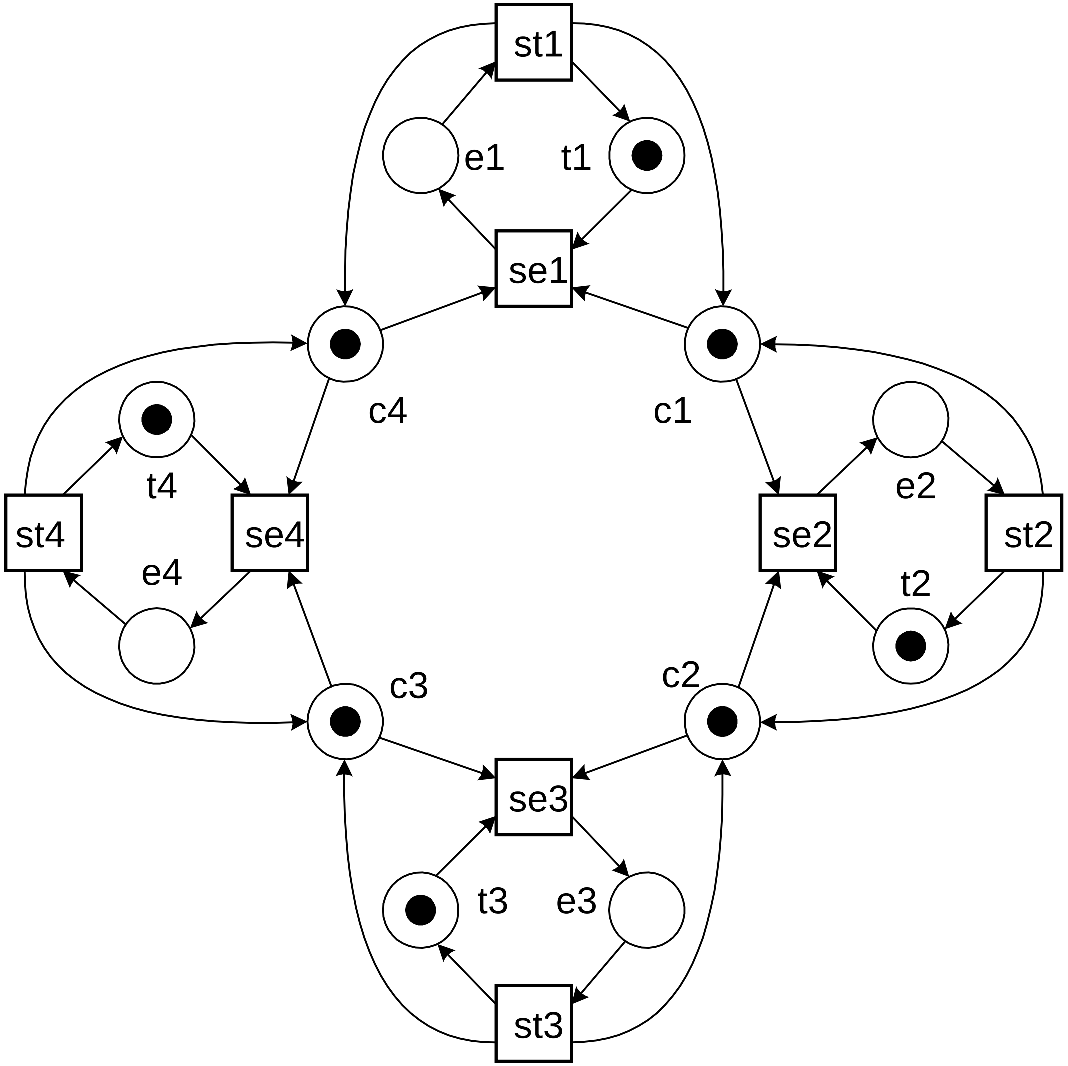


- rg1 + go1 + or1
- rg2 + go2 + or2
- rg1 + rg2 + go1 + go2 + or1 + or2
- 4 rg1 + 3 rg2 + 4 go1 + 3 go2 + 4 or1 + 3 or2

Exercise: Give transition invariants



Exercise: four philosophers



- **Give place invariants.**
- **Give transition invariants**

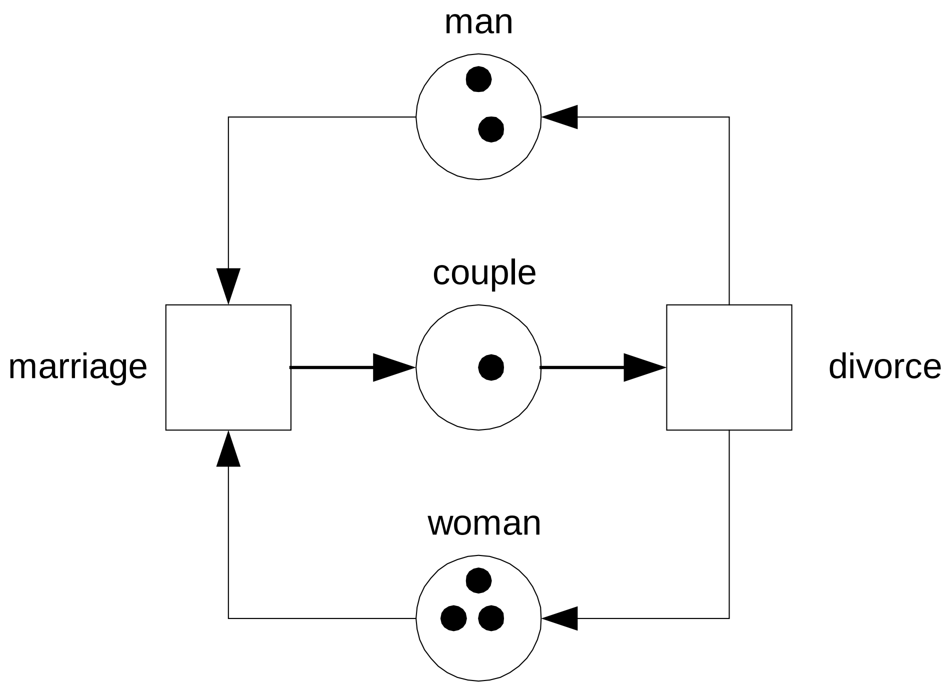
Two ways of calculating invariants

- **"Intuitive way": Formulate the property that you think holds and verify it.**
- **"Linear-algebraic way": Solve a system of linear equations.**

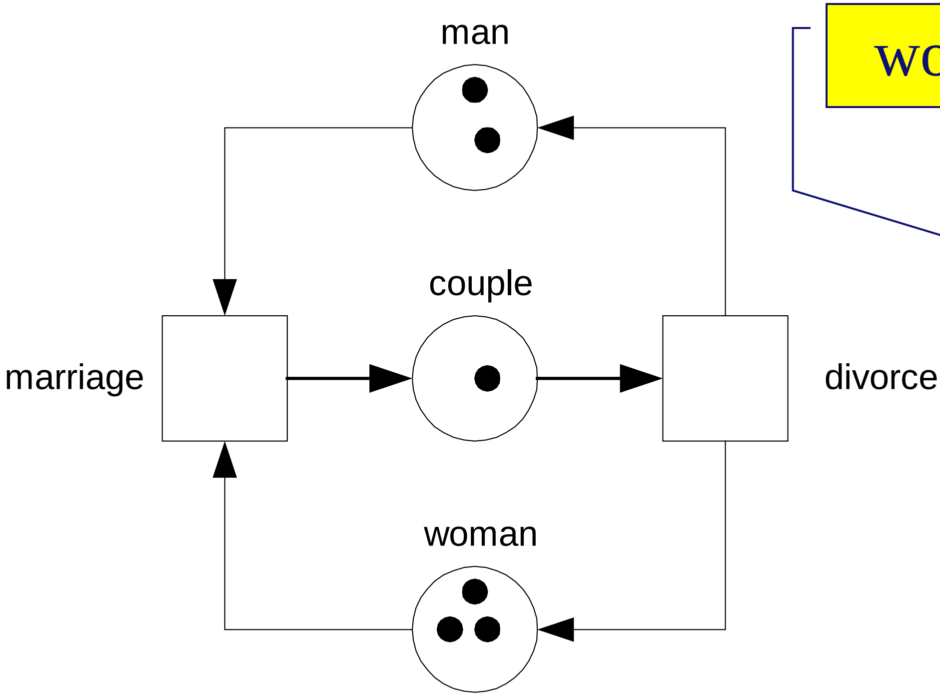
Humans tend to do it the intuitive way and computers do it the linear-algebraic way.

Incidence matrix of a Petri net: Old example

- **Each row corresponds to a place.**
- **Each column corresponds to a transition.**



$$N = \begin{pmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$



transitions

places

$$N = \begin{pmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

man

woman

couple

marriage

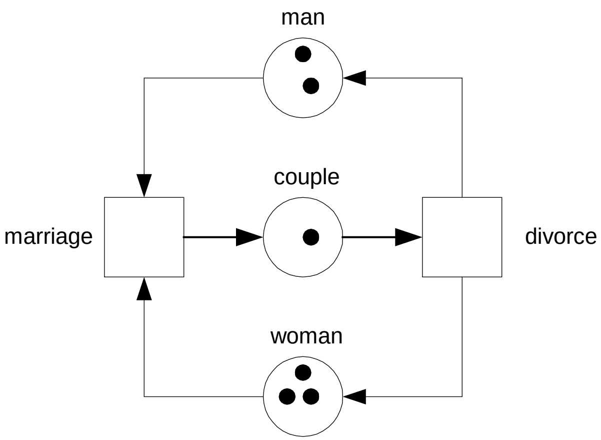
divorce

Place invariant

- **Let N be the incidence matrix of a net with n places (rows) and m transitions (columns), i.e., an n x m matrix.**
- **Any solution of the equation *X.N = 0* is a place invariant**
  - ***X* is a row vector (i.e., *1 x n* matrix)**
  - ***O* is a row vector (i.e., *1 x m* matrix)**
- **Note that (0,0,... 0) is always a place invariant.**
- **Basis can be calculated in polynomial time.**

Example

$$X\begin{pmatrix}-1 & 1\\ -1 & 1\\ 1 & -1\end{pmatrix} = (0,0)$$



$$(man, woman, couple)\begin{pmatrix}-1 & 1\\ -1 & 1\\ 1 & -1\end{pmatrix} = (0,0)$$

Solutions for X:

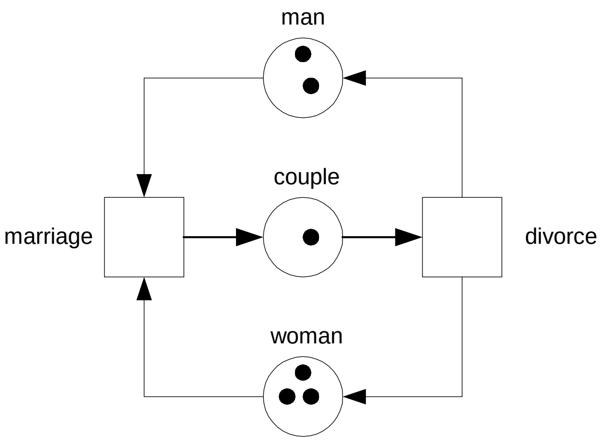
- **(0,0,0)**
- **(1,0,1)**
- **(0,1,1)**
- **(1,1,2)**
- **(1,-1,0)**

so man-1 + woman*-1 + couple *1 = 0 and man*1 + woman*1 + couple *-1 = 0*

Transition invariant

- **Let N be the incidence matrix of a net with n places and m transitions**
- **Any solution of the equation *N.X = 0* is a transition invariant**
  - ***X* is a column vector (i.e., *m x 1* matrix)**
  - ***0* is a column vector (i.e., *n x 1* matrix)**
- **Note that $(0,0,\ldots 0)^T$ is always a transition invariant.**
- **Basis can be calculated in polynomial time.**

Example



$$\begin{pmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

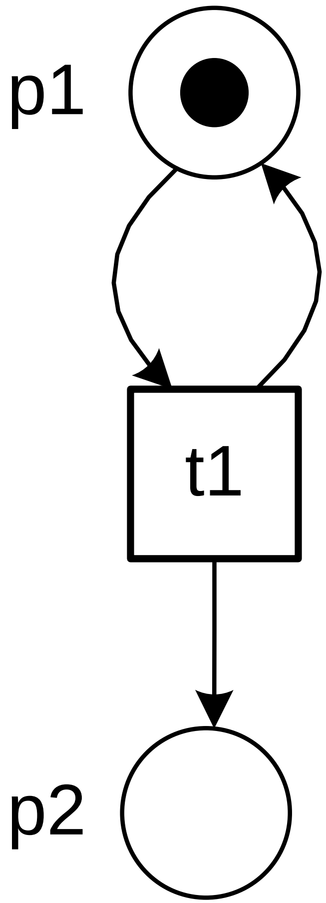
$$\begin{pmatrix} -1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} marriage \\ divorce \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Solutions:

- **$(0,0)^T$**
- **$(1,1)^T$**
- **$(32,32)^T$**

*so -1*marriage + 1*divorce = 0, -1*marriage + 1*divorce = 0, and 1*marriage + -1*divorce = 0*

Give place and transition invariants



place invariants

$$(p1, p2)\begin{pmatrix}0\\1\end{pmatrix} = (0)$$

*so p1*0 + p2*1 = 0*

- **Solutions:**
  - ❑ **(1,0) (i.e., p1)**
  - ❑ **(5,0) (i.e., 5 p1)**

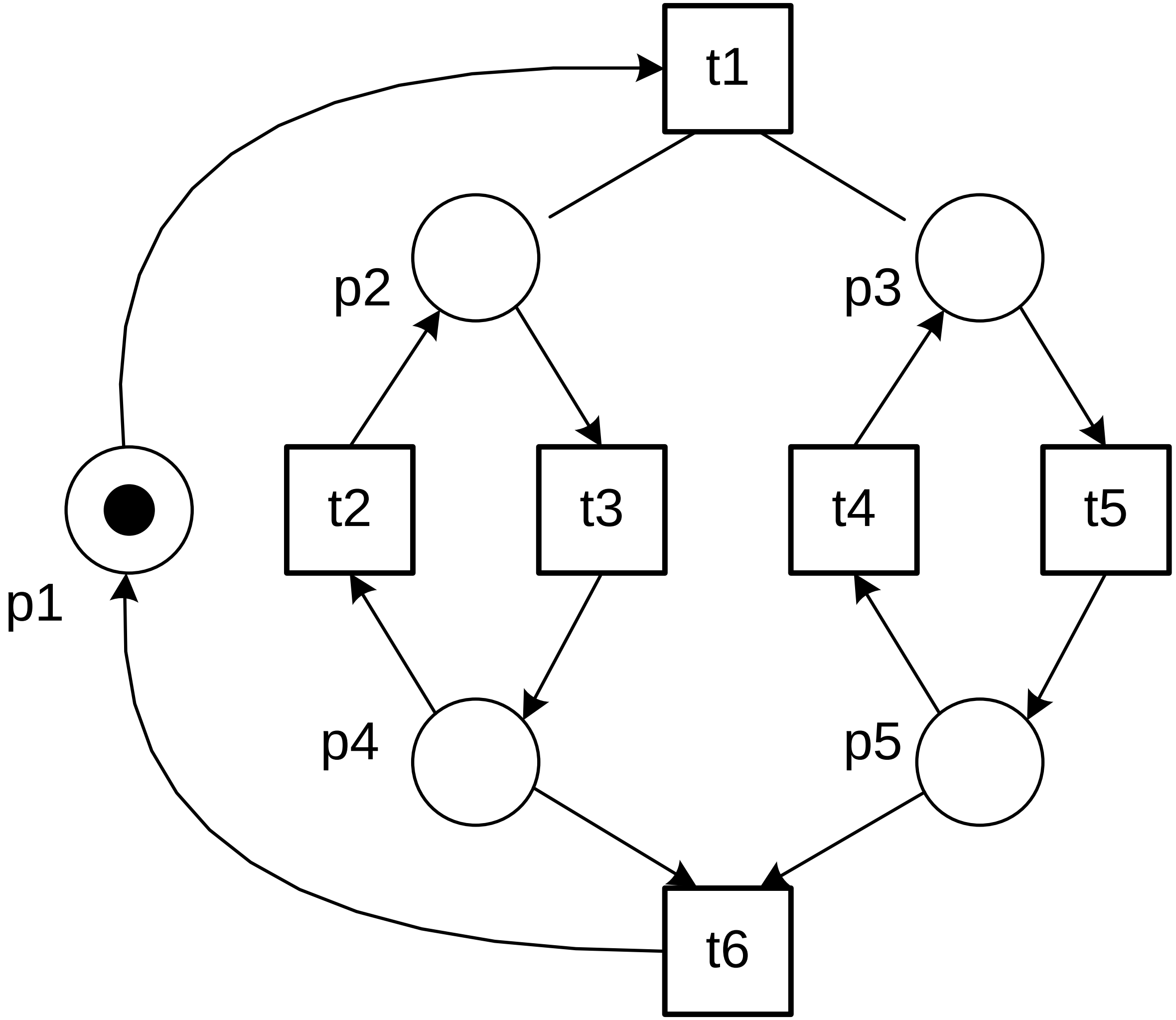
transition invariants

$$\begin{pmatrix}0\\1\end{pmatrix}(t1) = \begin{pmatrix}0\\0\end{pmatrix}$$

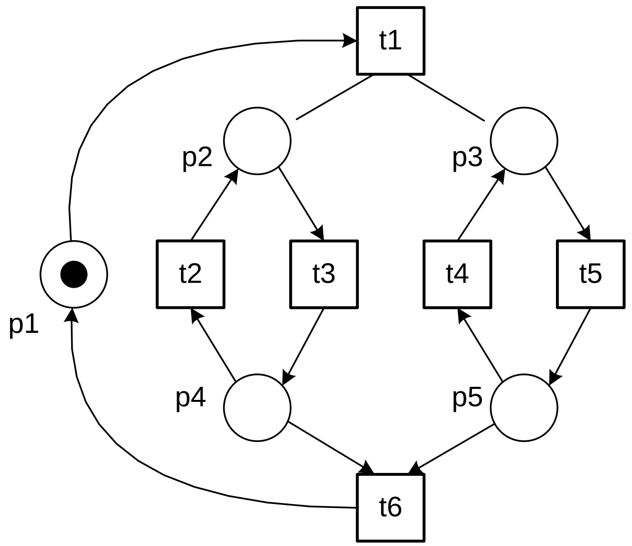
*so 0*t1 =0 and 1*t1 = 0*

- **Solutions:**
  - ❑ **(0) (i.e., 0 t1)**

Give place and transition invariants



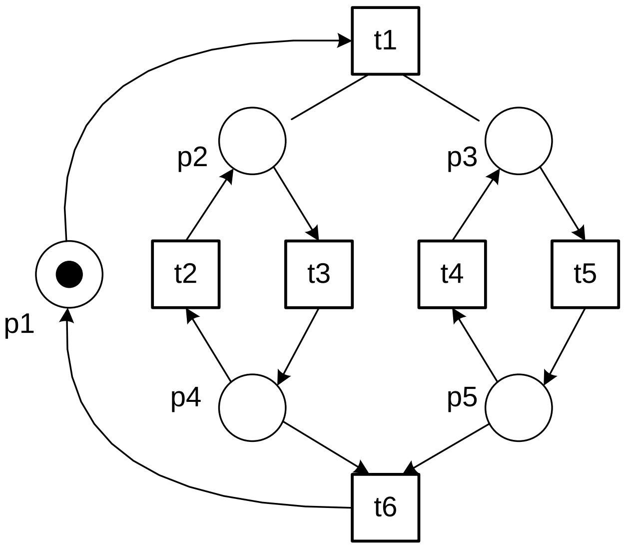
Place invariants



- **Solutions:**
  - **(1,1,0,1,0)   (i.e., p1+p2+p4)**
  - **(1,0,1,0,1)   (i.e., p1+p3+p5)**
  - **(2,1,1,1,1)   (i.e., 2 p1+p2+p3+p4+p5)**
  - **(6,5,1,5,1)**
  - **....**

$$(p1, p2, p3, p4, p5)\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{pmatrix} = (0,0,0,0,0,0)$$

Transition invariants



$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} t1 \\ t2 \\ t3 \\ t4 \\ t5 \\ t6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- **Solutions:**
  - **$(1,0,1,0,1,1)^T$ (i.e., t1+t3+t5+t6)**
  - **$(0,1,1,0,0,0)^T$ (i.e., t2+t3)**
  - **$(0,0,0,1,1,0)^T$ (i.e., t4+t5)**
  - **$(1,1,2,1,2,1)^T$ (i.e., t1+t2+2t3+t4+2t5+t6)**
  - **....**

Place invariants
(Formalization based on Desel and Reisig)

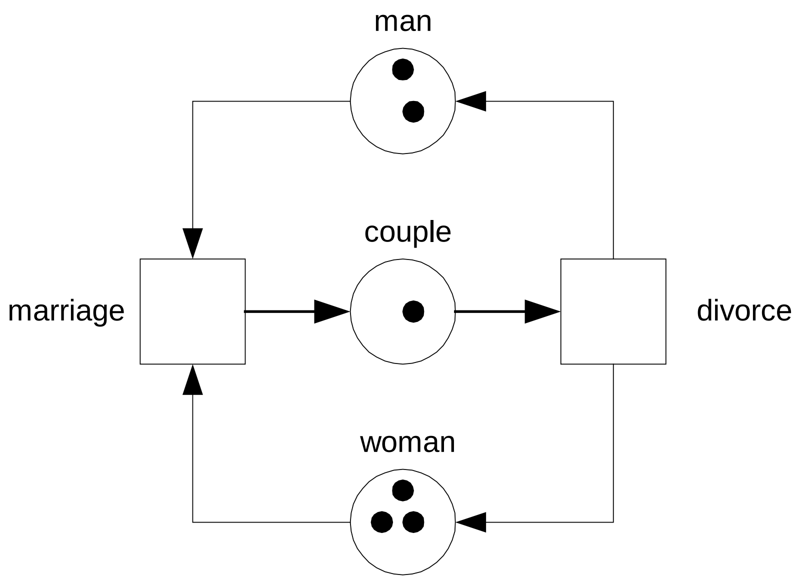
weight consumed

weight produced

Given a net N, a *place invariant* is a mapping $i : S_N \rightarrow \mathbb{Z}$ satisfying $i(s) \neq 0$ for finitely many places and

$$\sum_{s \in {}^\bullet t} i(s) = \sum_{s \in t^\bullet} i(s)$$

for each transition t of N. A place invariant is *nonnegative* if it maps no place to a negative number.



i(man)=1, i(couple)=1, i(woman) =0

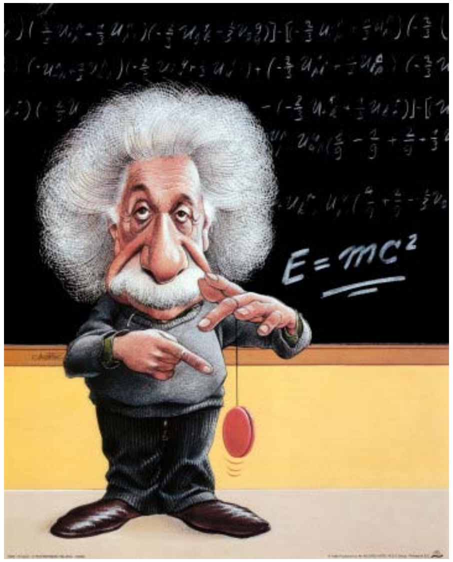
i(man)=1, i(couple)=2, i(woman) =1

i(man)=1, i(couple)=0, i(woman) =-1

Implication

Theorem 29. *If m is a reachable marking of a marked net with initial marking m_0 and i is a place invariant then*

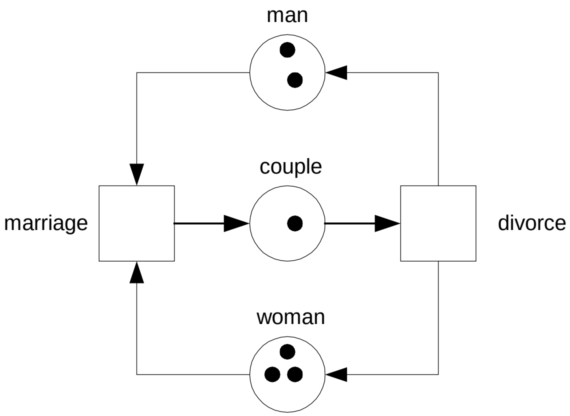
$$\sum_{s \in S_N} i(s) \cdot m(s) = \sum_{s \in S_N} i(s) \cdot m_0(s).$$

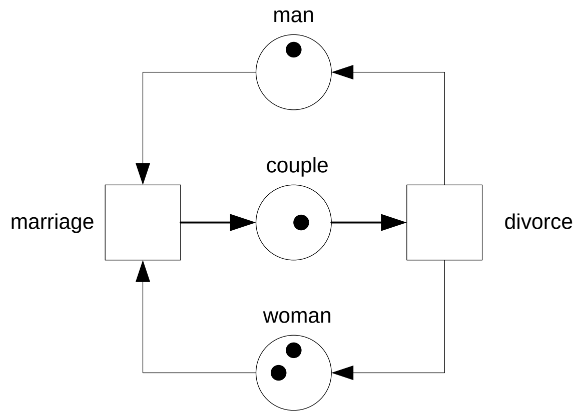


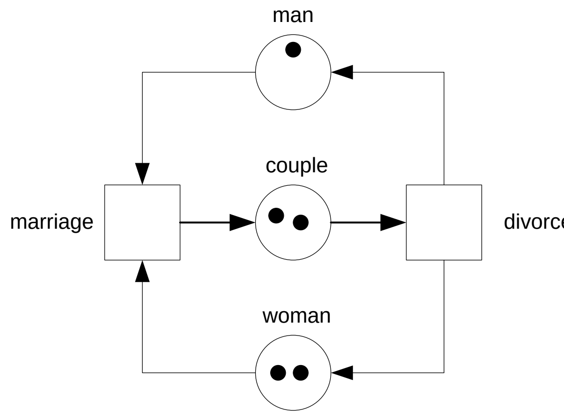
- **Conservation law!**
- **Can be used to show unreachability but not the reverse.**

Example

not reachable *reachable*







i(man)=1, i(couple)=1, i(woman) =0

1*2+1*1+0*3 = **3** 1*1+1*1+0*2 = **2** 1*1+1*2+0*2 = **3**

i(man)=1, i(couple)=2, i(woman) =1

1*2+2*1+1*3 = **7** 1*1+2*1+1*2 = **5** 1*1+2*2+1*2 = **7**

i(man)=1, i(couple)=0, i(woman) =-1

one direction only!

1*2+0*1+-1*3 = **-1** 1*1+0*1+-1*2 = **-1** 1*1+0*2+-1*2 = **-1**

Some properties

Theorem 30. *Assume a marked net N without dead transitions. Let m_0 be the initial marking. Let $i: S_N \to \mathbb{Z}$. If each reachable marking m satisfies*

$$\sum_{s \in S_N} i(s) \cdot m(s) = \sum_{s \in S_N} i(s) \cdot m_0(s)$$

then i is a place invariant.

Theorem 31. *Let s be a place of a marked net N with initial marking m_0. If there is a nonnegative place invariant i satisfying $i(s) \geq 1$ then s is bounded by*

$$\frac{1}{i(s)} \cdot \sum_{s' \in S_N} i(s') \cdot m_0(s').$$

move all weight to *s* …

Some more properties

Corollary 32. *A finite marked net is bounded if it has a place invariant i that maps all places to positive numbers.*

"if", not "if and only if" …

Theorem 33. *Let N be a marked net with initial marking m_0 and let i be a nonnegative place invariant. Let S_i be the set of places s satisfying $i(s) > 0$. If $m_0(s) = 0$ for each place in S_i then every transition in ${}^\bullet S_i \cup S_i^\bullet$ is dead at the initial marking.*

remains empty …

Transition invariants

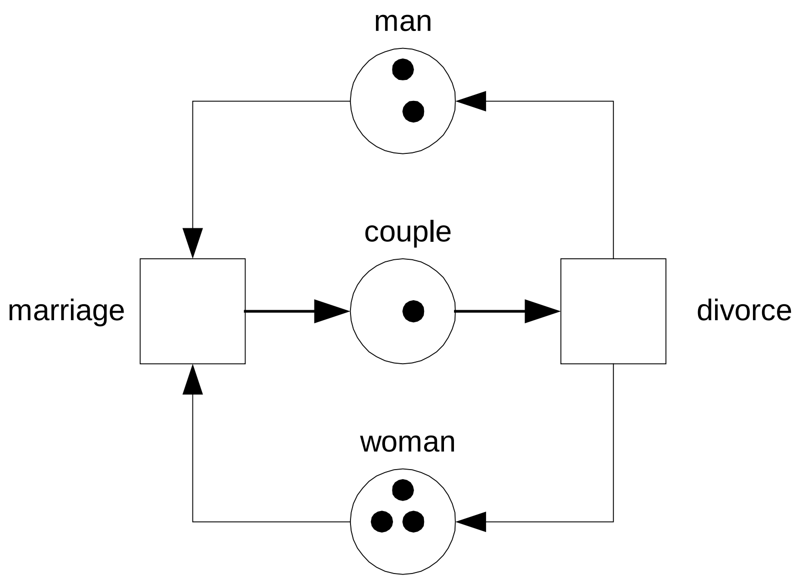
tokens produced for s

tokens consumed from s

Given a net N, a *transition inv*… *nt* is a mapping j: T … $\mathbb{Z}$ satisfying $j(t) \neq 0$ for finitely many transitions and …

$$\sum_{t \in {}^{\bullet}s} j(t) = \sum_{t \in s^{\bullet}} j(t)$$

for each place s of N.



j(marriage)=1, j(divorce)=1

j(marriage)=5, j(divorce)=5

Some properties

Let σ be a finite sequence of transitions of a net N. The *Parikh mapping* $p_\sigma : T_N \to \mathbb{Z}$ maps each transition t to the number of occurrences of t in σ.

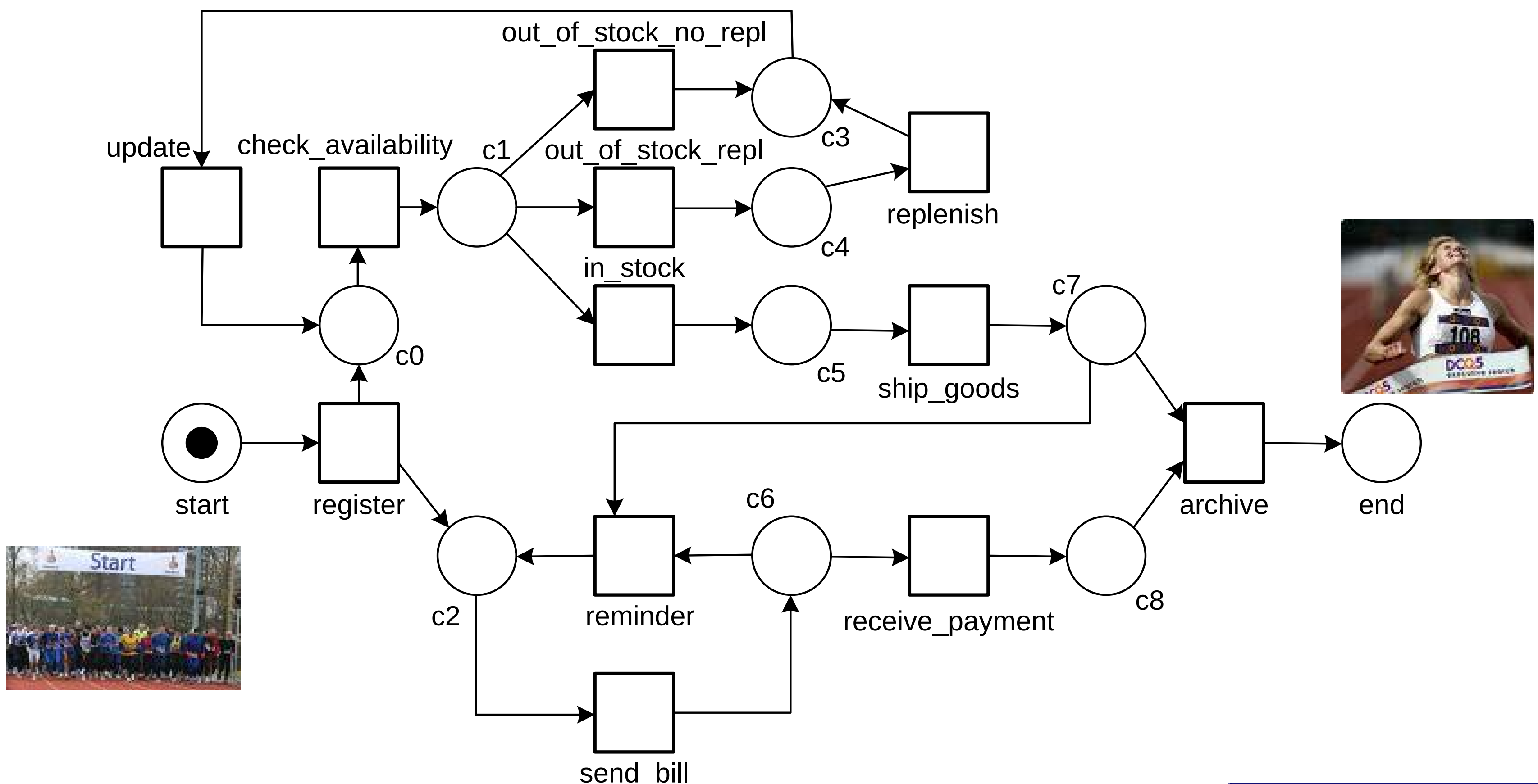
Theorem 35. *If $m \xrightarrow{\sigma} m'$ is a finite occurrence sequence of a net then $m = m'$ if and only if the Parikh mapping p_σ is a transition invariant.*

Corollary 37. *If a finite marked net is live and bounded then it has a transition invariant that maps each transitions to a positive number.*

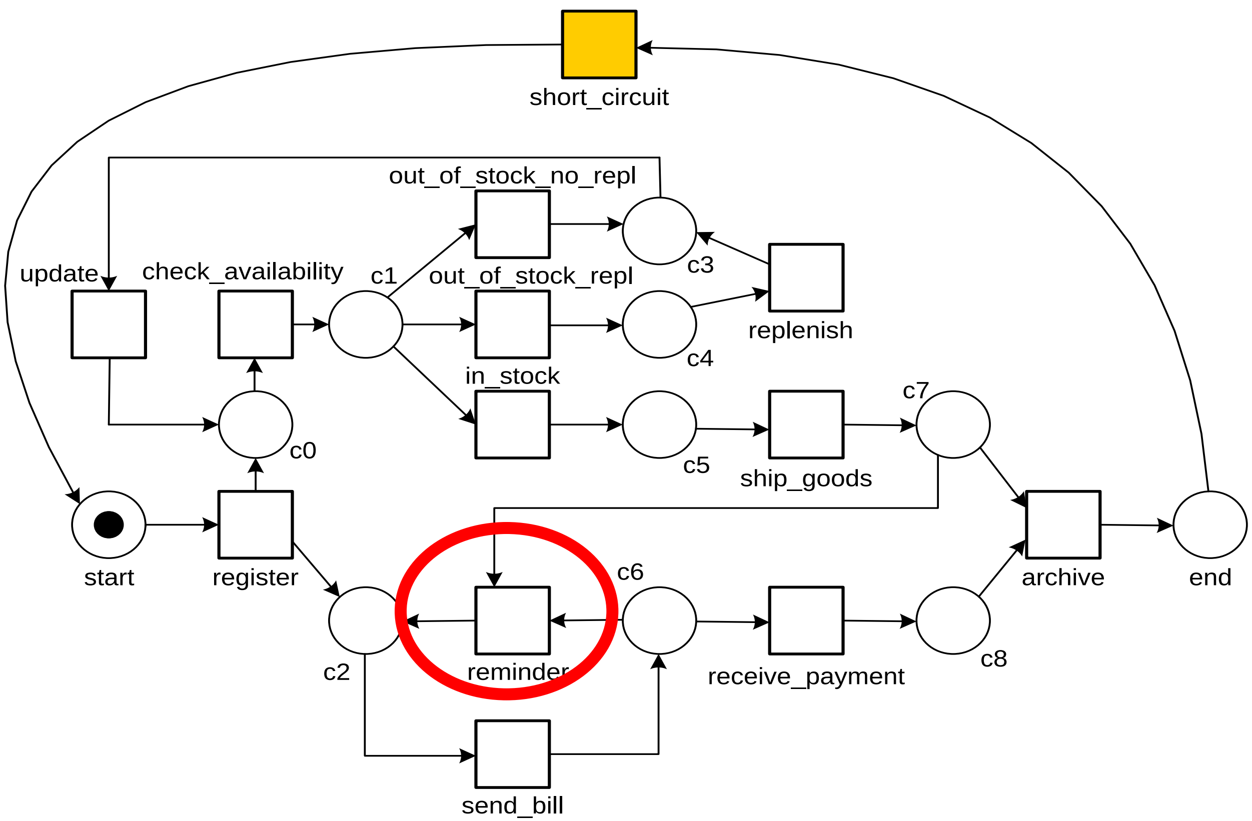
Typical scenario

1. **Make model cyclic (if needed)**
2. **If particular transitions are not covered by any positive transition invariants, then this reveals a possible problem.**

Correct?

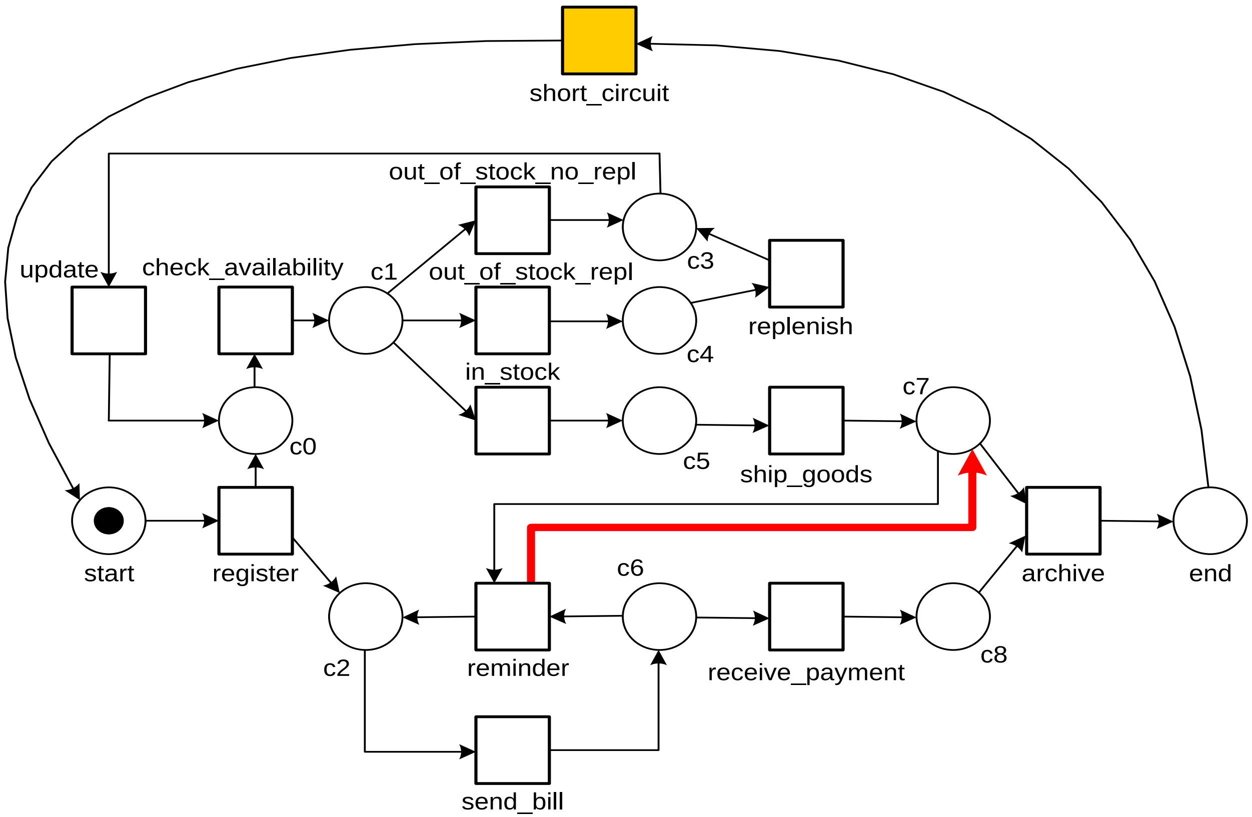

out_of_stock_no_repl
update
check_availability
c1
out_of_stock_repl
c3
replenish
c4
in_stock
c7
c0
c5
ship_goods
start
register
archive
end
c6
Start
c2
reminder
receive_payment
c8
send_bill

Short-circuit



not covered by any positive transition invariant

Repair



now, all transitions are covered by a positive transition invariant

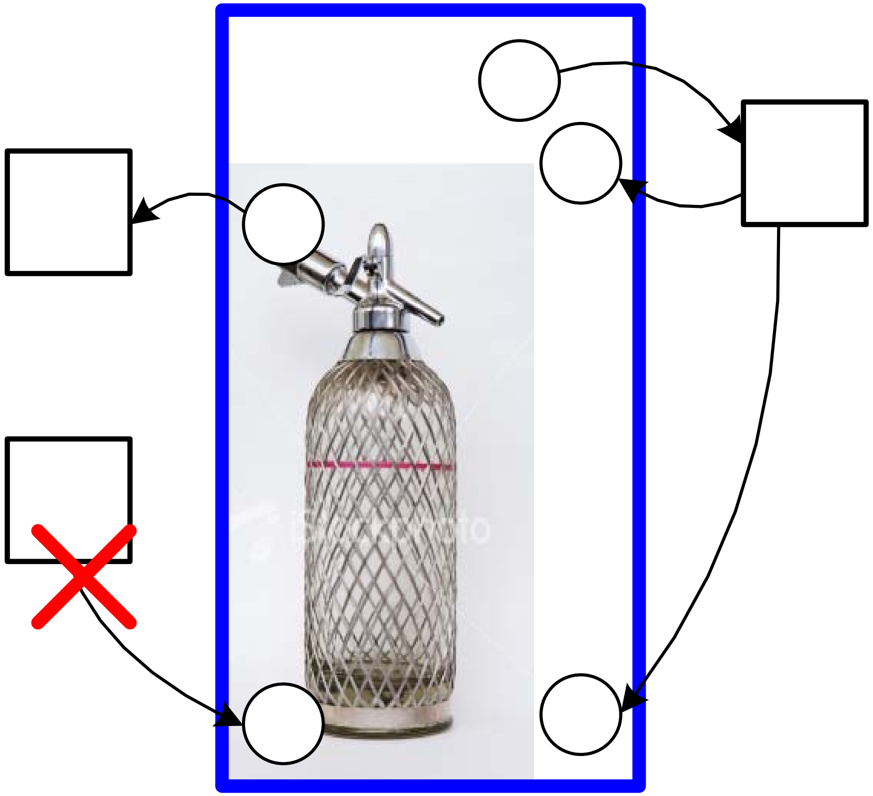
Siphons and Traps

Technische Universiteit
Eindhoven
University of Technology

Where innovation starts

Siphon
(Formalization based on Desel and Reisig)

A *siphon* is a set S of places satisfying $^\bullet S \subseteq S^\bullet$. A siphon is *marked* by a marking m if at least one place of it is marked at m.



"transitions that add a token to the siphon also remove a token"

Behavior of siphons: Once unmarked always unmarked

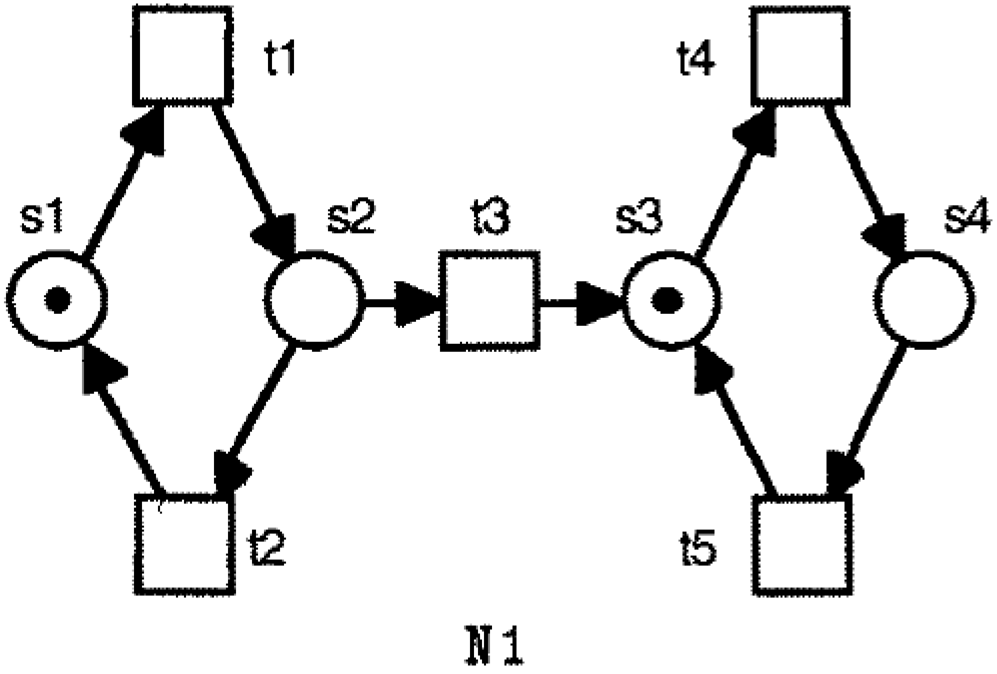
Theorem 38. *Assume a marked net with a siphon S. If S is not marked at the initial marking then S is not marked at any reachable marking.*

Proof. We apply Lemma 28 to show that every reachable marking marks no place of S.

Let M be the set of markings that do not mark S. By assumption, the initial marking is in M. Assume a marking m in M and a transition occurrence $m \xrightarrow{t} m'$. Then $t \notin S^{\bullet}$ because m enables t and m marks no place in S. Since S is a siphon, this implies $t \notin {}^{\bullet}S$. Hence no place of S can gain a token by the occurrence of t and m' belongs to M, too.

So, by Lemma 28, M includes all reachable markings, which implies the result. □

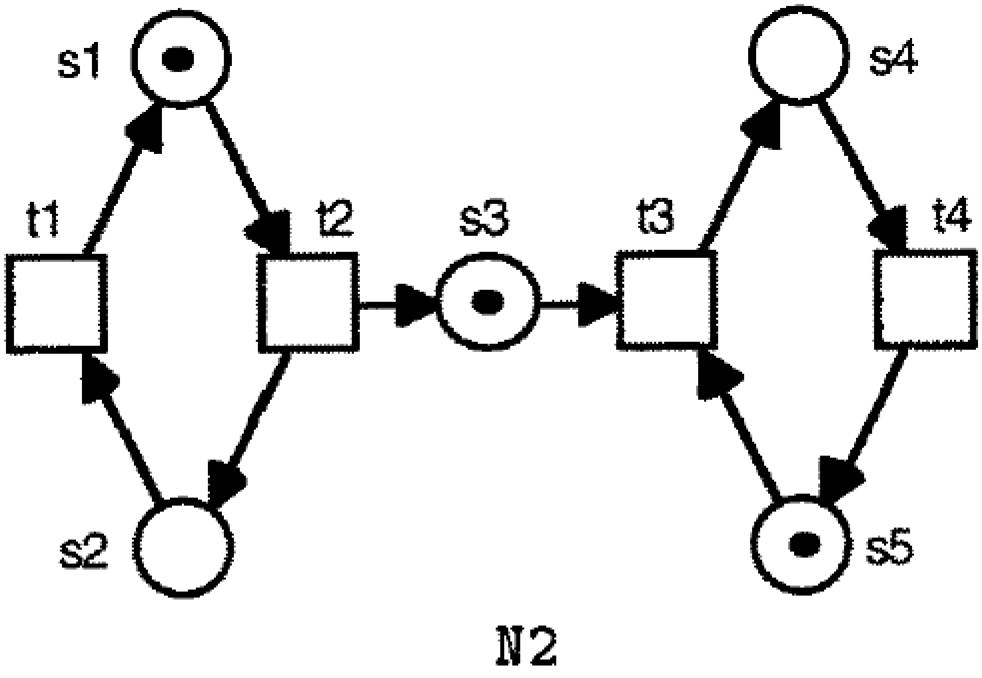
Siphons



"transitions that add a token to the siphon also remove a token"

$$\emptyset, \{s1, s2\}, \{s1, s2, s3, s4\}$$

Siphons



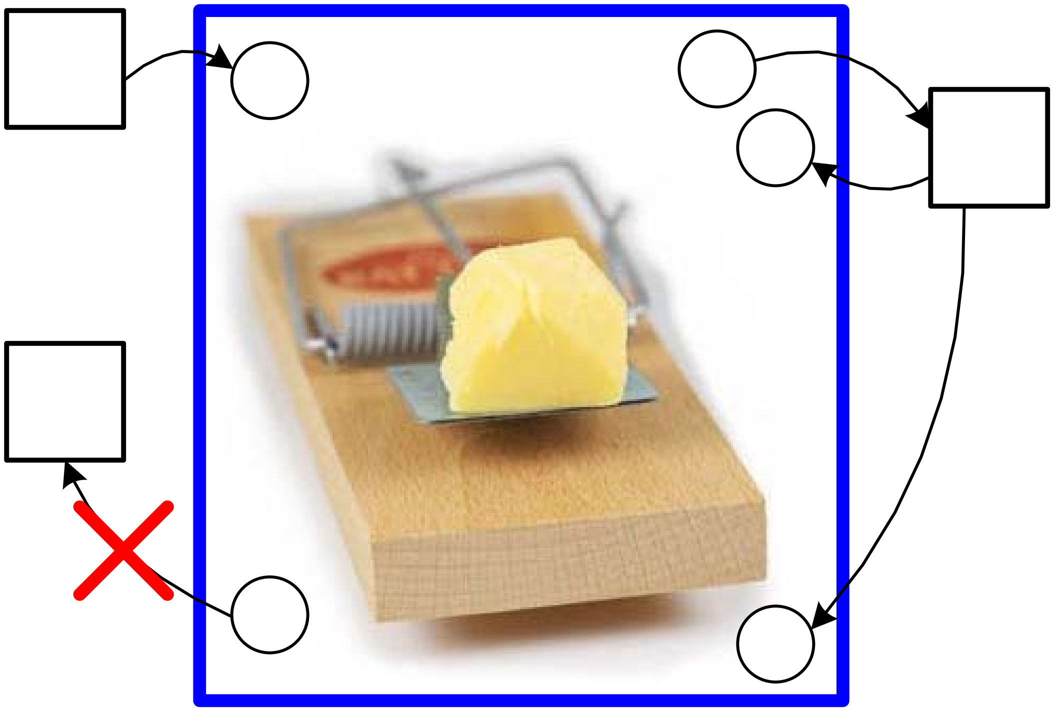
"transitions that add a token to the siphon also remove a token"

the union of some siphons is again a siphon

$\emptyset, \{s1, s2\}, \{s1, s2, s3\}, \{s1, s2, s3, s4\}, \{s1, s2, s3, s4, s5\}, \{s4, s5\}$

Trap

A *trap* is a set S of places satisfying $S^\bullet \subseteq {}^\bullet S$. A trap is *marked* by a marking m if at least one place of it is marked at m.



"transitions that remove a token from the trap also add a token"

Behavior of traps: Once marked always marked

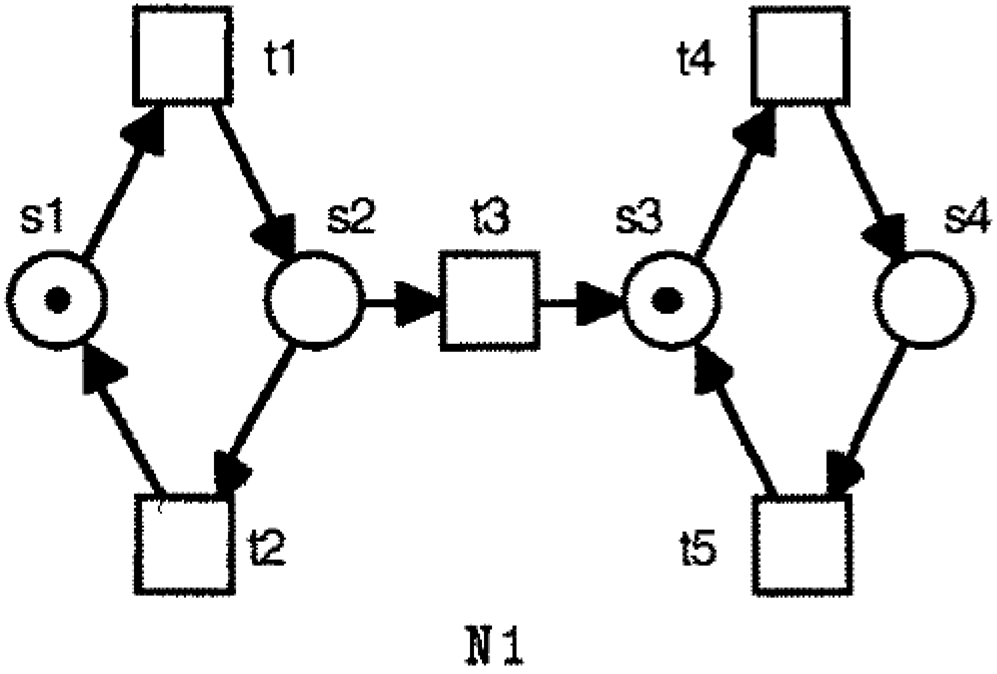
Theorem 40. *Assume a marked net with a trap S. If S is marked at the initial marking then it is marked at every reachable marking.*

Proof. We apply Lemma 28.

Let M be the set of markings of the net that mark at least one place of S. By assumption, the initial marking is in M. Now assume a marking m in M and a transition occurrence $m \xrightarrow{t} m'$. If $t \notin S^\bullet$ then the place of S marked by m remains marked. If $t \in S^\bullet$ then $t \in {}^\bullet S$ because S is a trap. Hence, in this case at least one place in $t^\bullet \cap S$ is marked at m'.

So, by Lemma 28, M includes all reachable markings, which implies the result. □

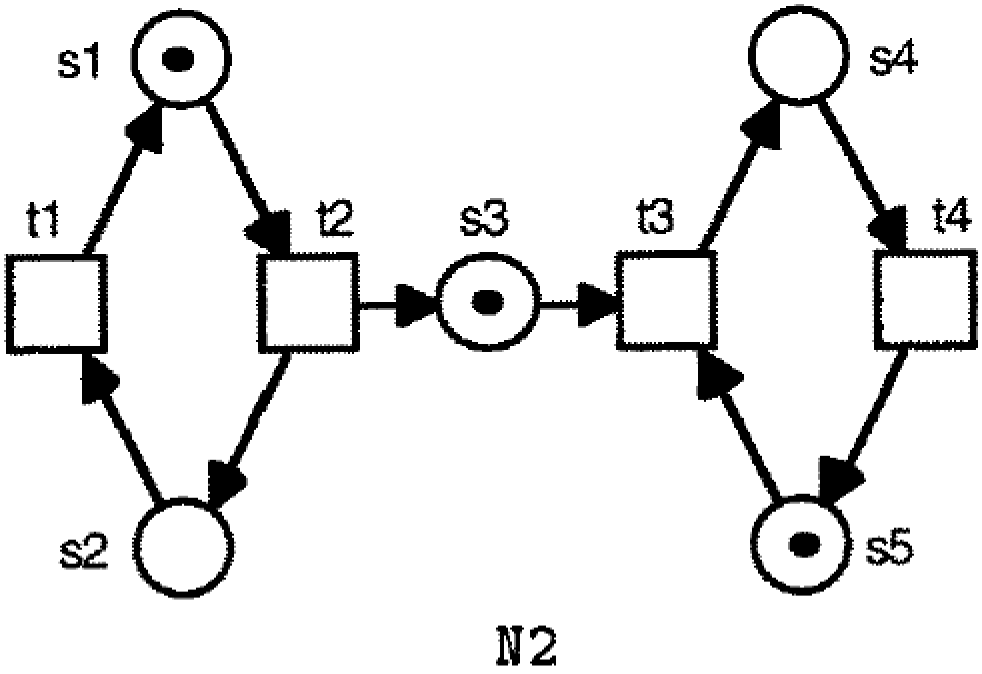
Traps



"transitions that remove a token from the trap also add a token"

$\emptyset, \{s3, s4\}, \{s1, s2, s3, s4\}$

Traps



"transitions that remove a token from the trap also add a token"

the union of some traps is again a trap

$\emptyset, \{s1, s2\}, \{s1, s2, s3, s4, s5\}, \{s2, s3, s4, s5\}, \{s3, s4, s5\}, \{s4, s5\}$

find error (also in paper)

Main Theorem: Sufficient condition for deadlock-freedom

Theorem 42. *Assume a marked net with at least one transition. If each non-empty siphon without isolated places includes a trap marked at the initial marking then the marked net is deadlock-free.*

Proof. Assume that the marked net is not deadlock-free and let m be a dead reachable marking. Let S be the set of non-isolated places that are not marked at m. We show that S is a non-empty siphon that includes no initially marked trap.

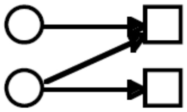
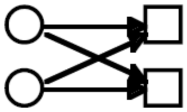
Each transition t is dead at m and hence has an unmarked input place. So $S^\bullet$ contains the set of all transitions. Therefore, ${}^\bullet S \subseteq S^\bullet$. S is not empty because the net has some transition by assumption and S contains a place in the pre-set of this transition. So S is a non-empty siphon without isolated places. By definition, S is not marked at m. Hence, S includes no trap marked at m. Since initially marked traps remain marked, S includes no trap marked at the initial marking. □

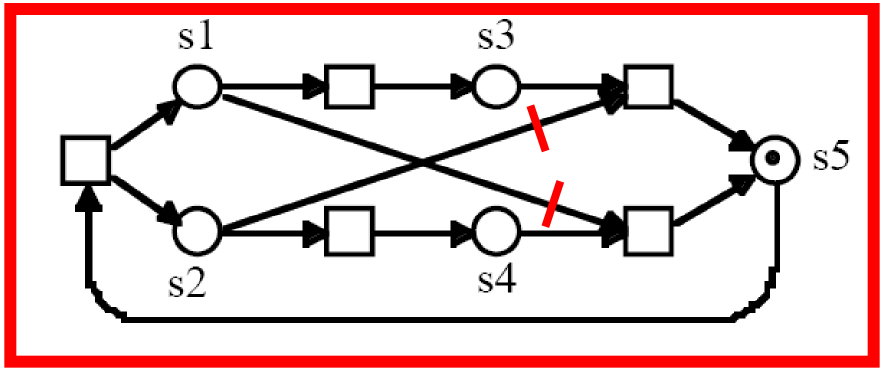
If every proper siphon of a system includes an initially marked trap, then the system is deadlock free.

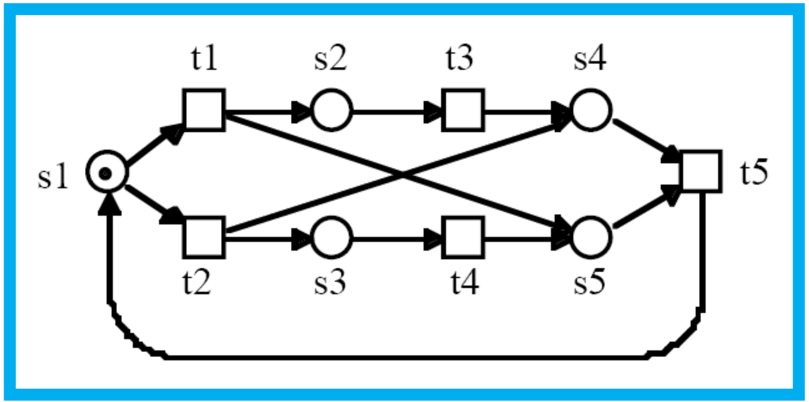
all siphons ⇒ some transition is enabled at any time

Intermezzo: Commoner's Theorem (1972)

A free-choice system is live if and only if every proper siphon includes an initially marked trap.

If  then 





Conclusion

TU/e Technische Universiteit Eindhoven University of Technology

Where innovation starts

After this lecture you should be able to:

- Determine whether a transition in a marked net is impartial, fair, or just, both by hand and by using CPN Tools.
- Construct nets that have transitions that satisfy certain fairness properties, e.g., a net containing impartial, fair, and just transitions.
- Understand the importance and role of invariants.
- Provide meaningful place invariants for a given net.
- Provide meaningful transition invariants for a given net.
- Understand the matrix representation of nets and invariants.
- Understand siphons and traps.
- Provide siphons and traps for a given net.
- Derive conclusions from the presence or absence of particular siphons and traps (e.g., deadlock-freedom).