

Analysis of Process Models: Introduction, state space analysis and simulation in CPN Tools

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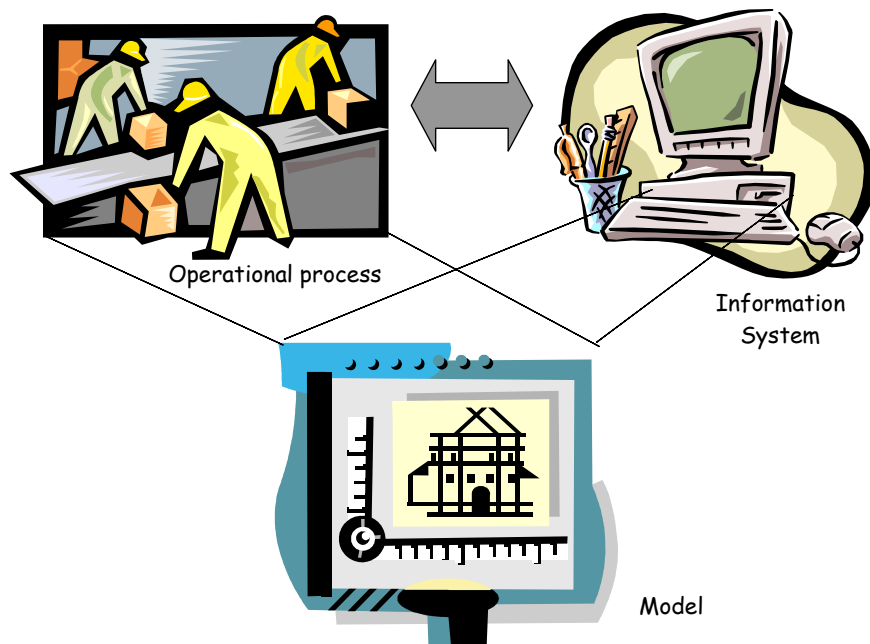
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Where innovation starts

What is a Petri net?

- A graphical notion (model = picture?)
 - A mathematical notion (model = graph?)
 - A programming notion (model = program?)
-
- **A solver independent medium**
 - **Starting point for a variety of analysis approaches**

Analysis



- Analysis is typically model-driven to allow e.g. what-if questions.
- Models of both operational processes and/or the information systems can be analyzed.
- Types of analysis:
 - *validation*
 - *verification*
 - *performance analysis*

Three types of analysis techniques

1. Reachability/coverability graph

2. Structural techniques

- Place and transition invariants
- Marking equation
- Traps, siphons, etc.

3. Simulation

- Each can be applied to both classical and high-level Petri nets.
- Nevertheless, for the second we restrict ourselves to classical Petri nets.

Mapping technique/use:

- **reachability graph** (validation, verification)
- **invariants** (validation, verification)
- **simulation** (validation, performance analysis)

Informal introduction ...

TU/e

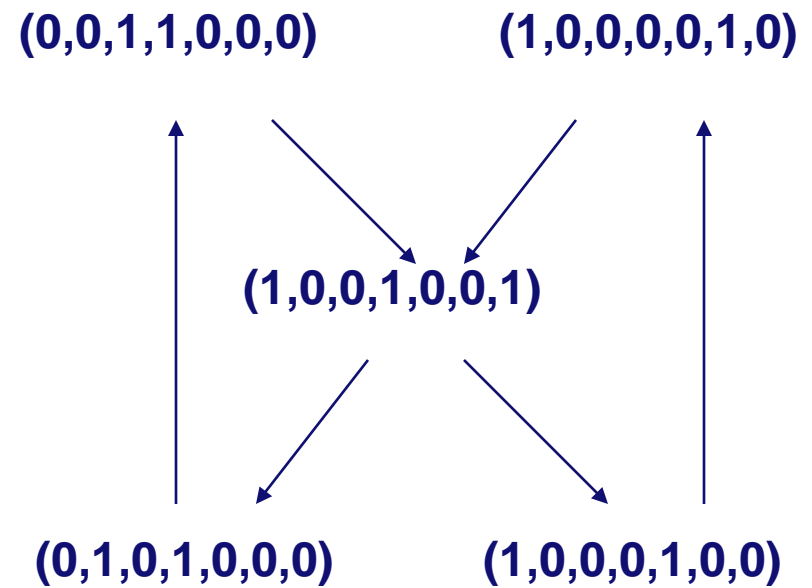
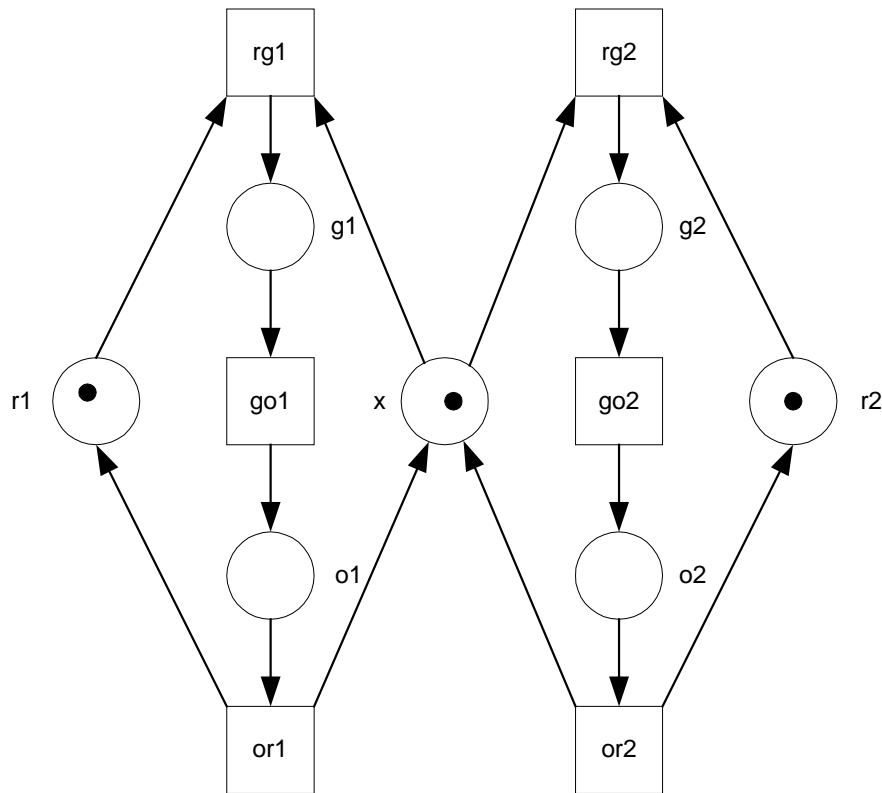
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Where innovation starts

Examples of generic questions given a marked Petri net







- **terminating**
it has only finite occurrence sequences
- **deadlock-free**
each reachable marking enables a transition
- **live**
each reachable marking enables an occurrence sequence containing all transitions
- **bounded**
each place has an upper bound that holds for all reachable markings
- **1-safe**
1 is a bound for each place p
- **reversible**
 m_0 is reachable from each reachable marking, i.e., the initial marking is a so-called home marking.

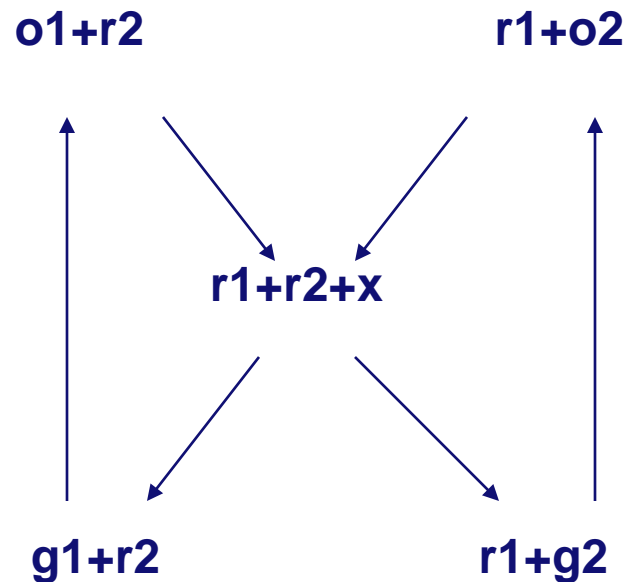
Reachability graph



**Five reachable states.
Traffic lights are safe!**

Alternative notation

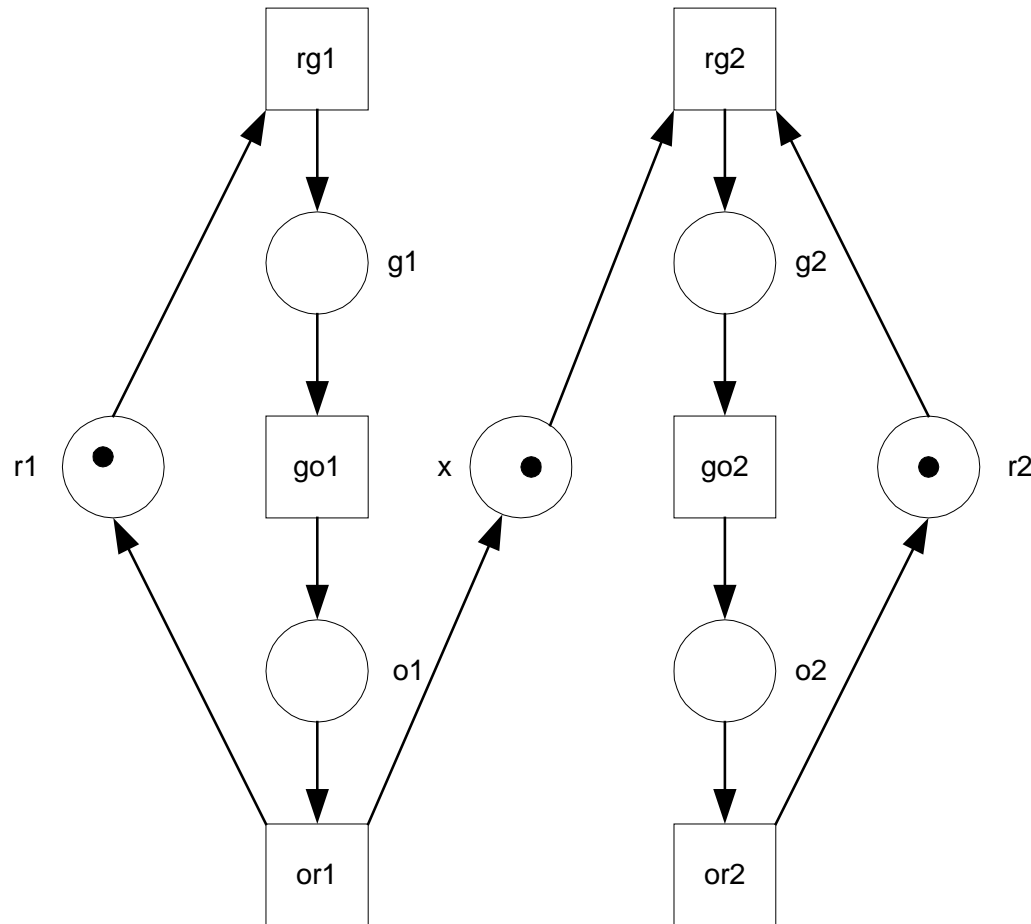
- **terminating**
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Reachability graph (2)

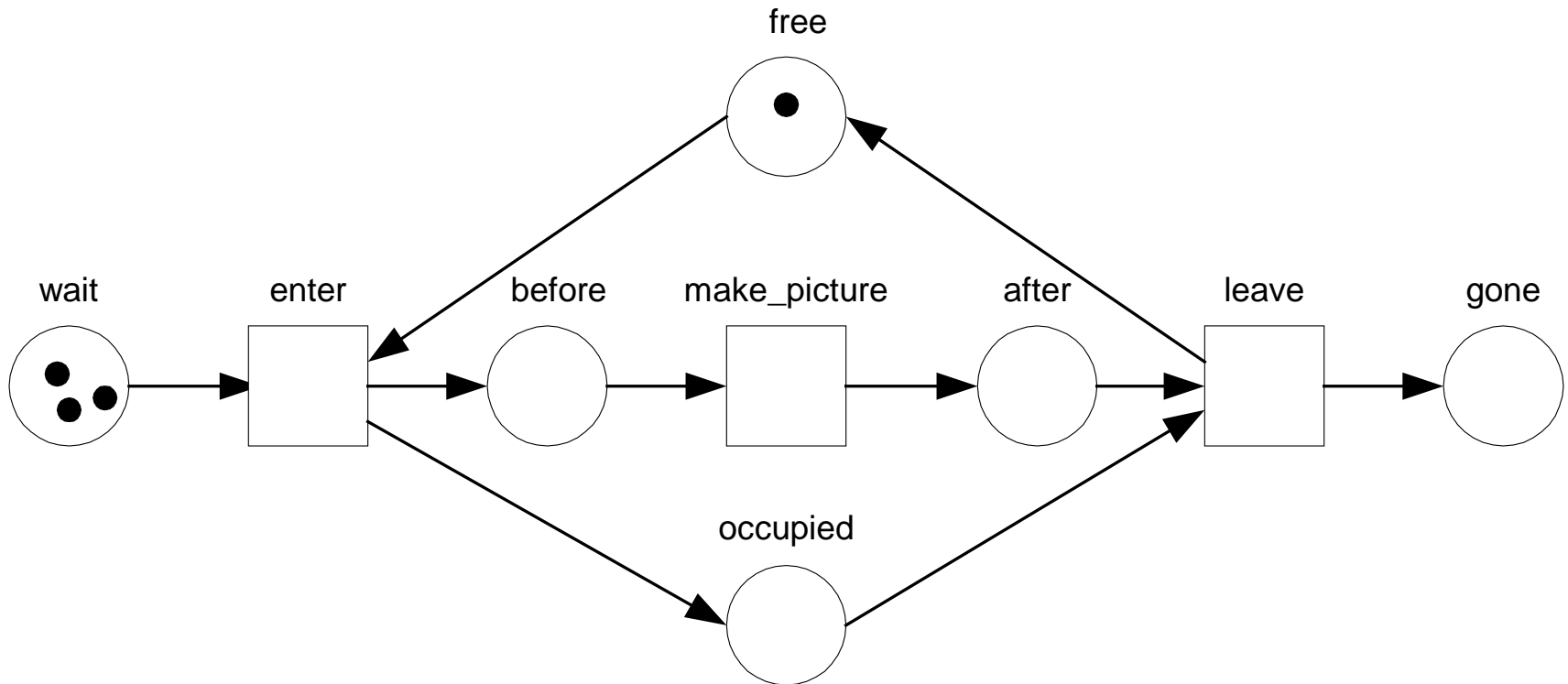
- Graph containing a node for each reachable state.
- Constructed by starting in the initial state, calculate all directly reachable states, etc.
- Expensive technique.
- Only feasible if finitely many states (otherwise use coverability graph).
- Difficult to generate diagnostic information.

Infinite reachability graph



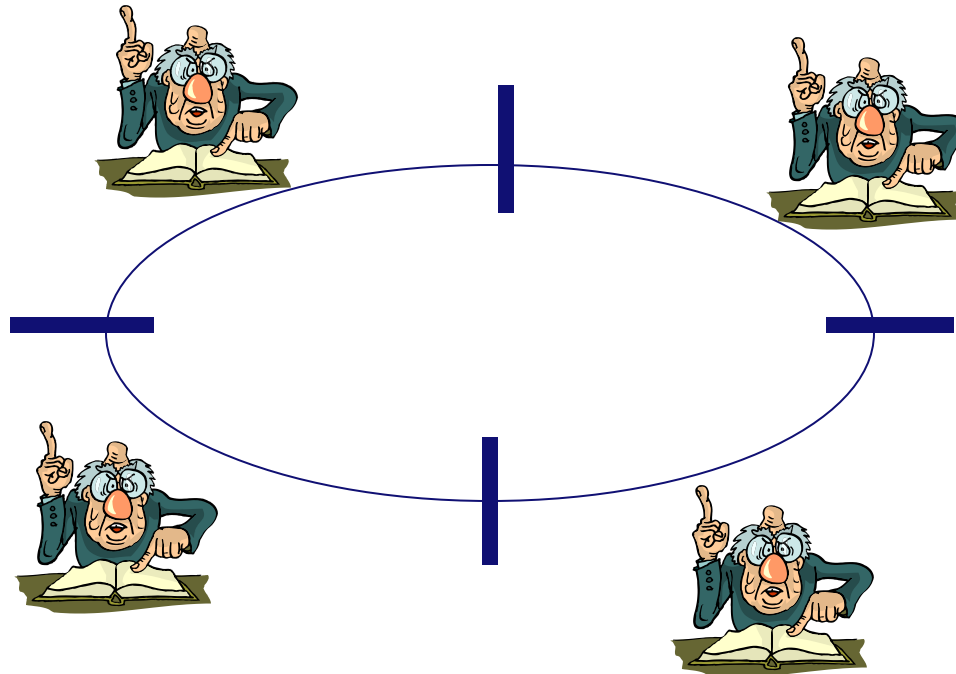
Therefore tools use a coverability graph which is always finite!

Exercise: Construct reachability graph



Exercise: Dining philosophers

- 4 philosophers sharing 4 chopsticks: A philosopher is either in state eating or thinking and needs two chopsticks to eat.
- Model as a Petri net and construct reachability graph.



See also: www.workflowcourse.com


Reachability graph:
The philosophers

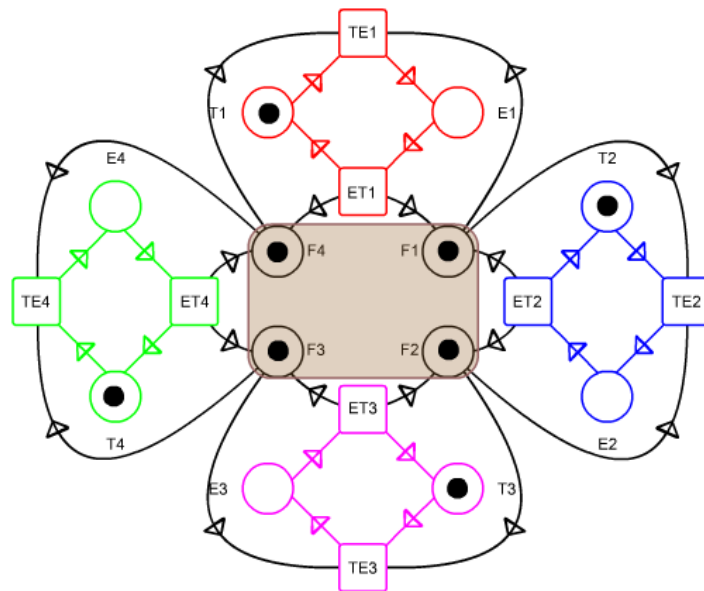
Model 1

Model 2

Model 3

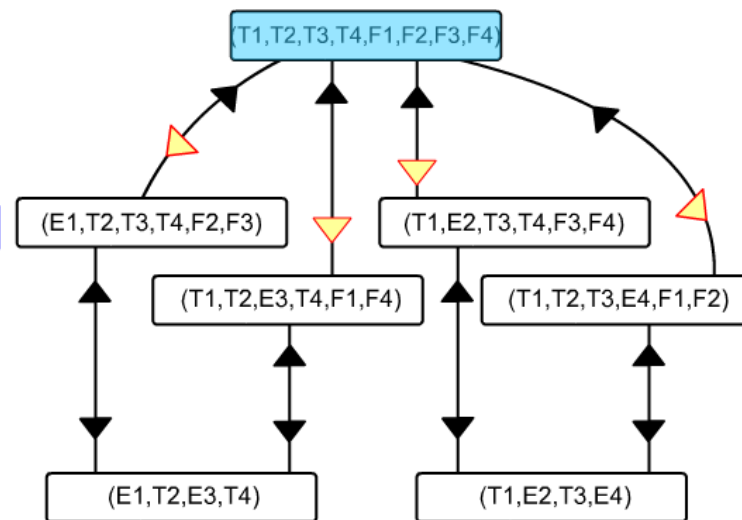
Menu

Press: 



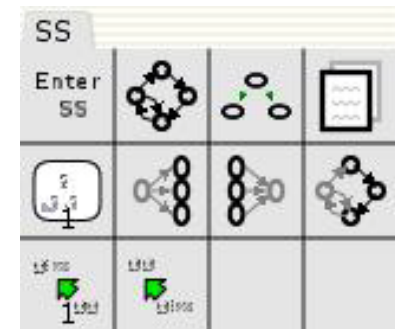
T: Thinking
TE: Finished thinking, Start eating
E: Eating
ET: Finished eating, Start thinking
F: Fork

Vector = (Position of the tokens)

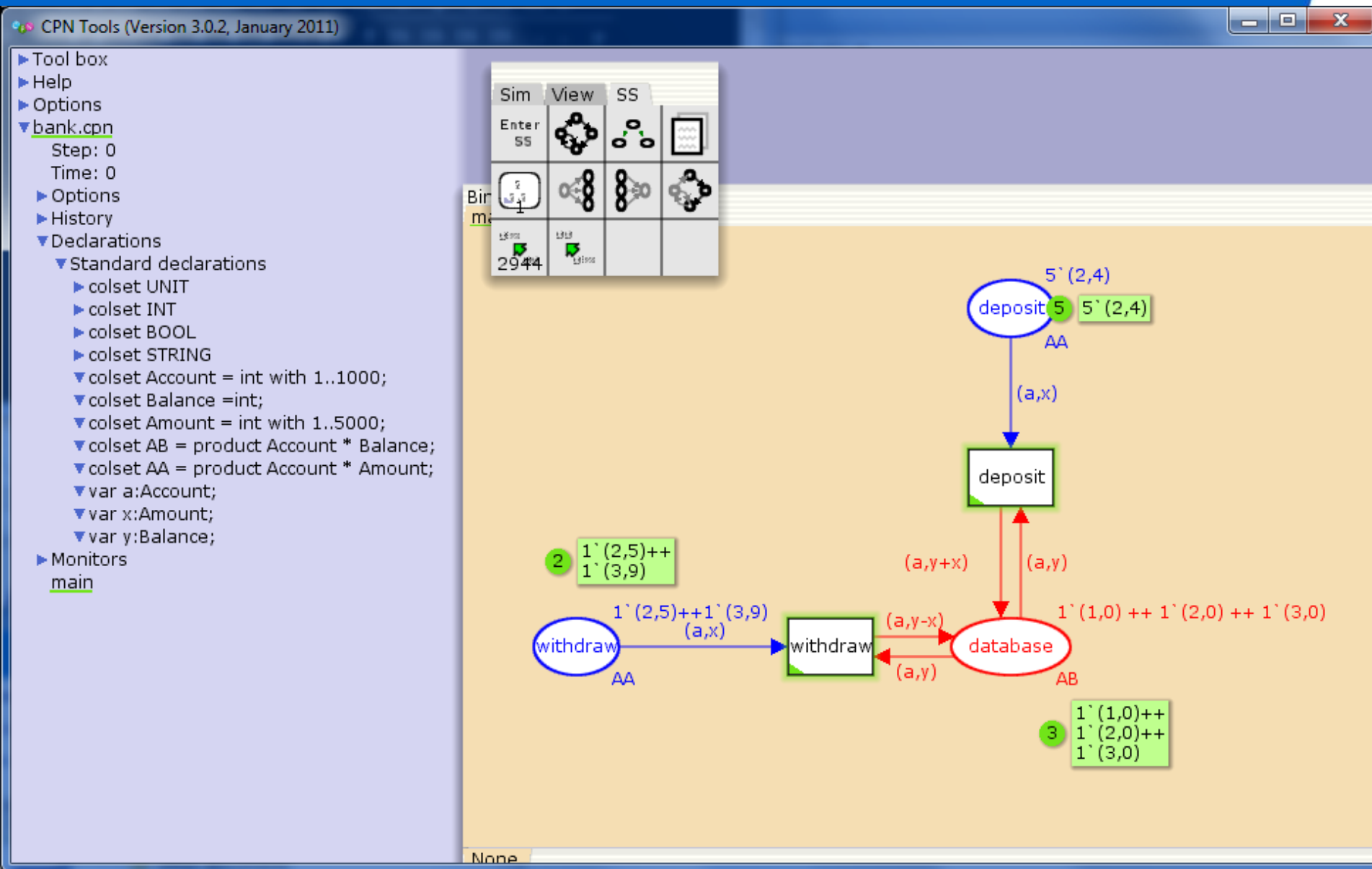


Analysis in CPN Tools

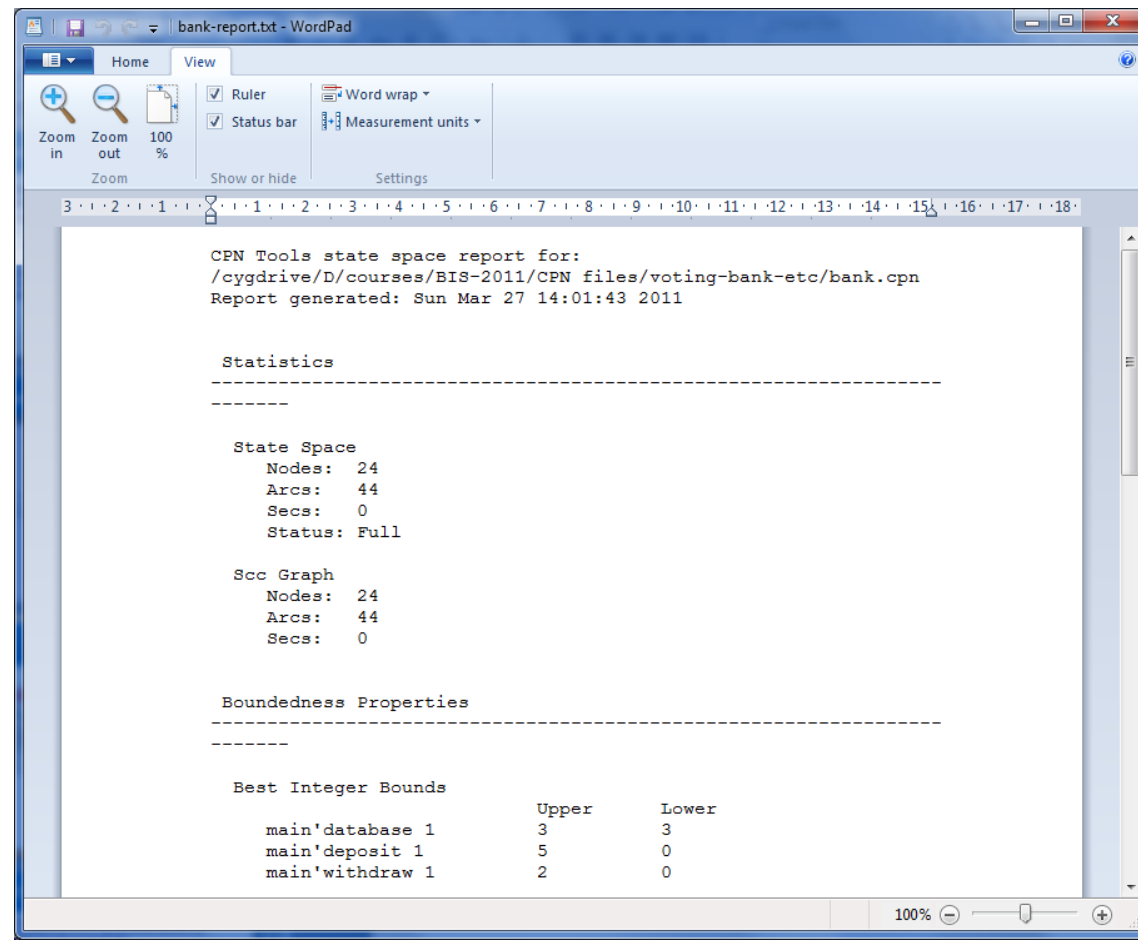
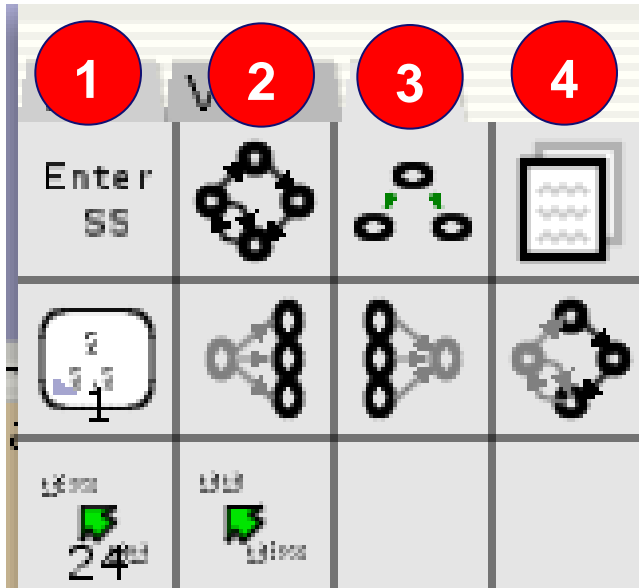
- Only state-space analysis, i.e., no invariants.
- Generate report in text file.
- State-space visualization from version 2.2.
- Steps:
 1. Enter the State Space Tool (to generate ML code)
 2. Calculating the state space
 3. Calculating the SCC graph
(to calculate home states and fairness)
 4. Save/view state space report



Example



Create report



Report (1)

CPN Tools state space report for:
/cygdrive/D/courses/BIS-2011/CPN files/voting-bank-etc/bank.cpn
Report generated: Sun Mar 27 14:01:43 2011

Statistics

State Space

Nodes: 24
Arcs: 44
Secs: 0
Status: Full

Scc Graph

Nodes: 24
Arcs: 44
Secs: 0

Report (2)

Boundedness Properties

Best Integer Bounds

	Upper	Lower
main'database 1	3	3
main'deposit 1	5	0
main'withdraw 1	2	0

Best Upper Multi-set Bounds

main'database 1 $1^{\sim}(1,0)++1^{\sim}(2,(\sim 5))++1^{\sim}(2,(\sim 1))++1^{\sim}(2,0)++1^{\sim}(2,3)++1^{\sim}(2,4)++1^{\sim}(2,7)++1^{\sim}(2,8)++1^{\sim}(2,11)++1^{\sim}(2,12)++1^{\sim}(2,15)++1^{\sim}(2,16)+1^{\sim}(2,20)++1^{\sim}(3,(\sim 9))++1^{\sim}(3,0)$

main'deposit 1 $5^{\sim}(2,4)$

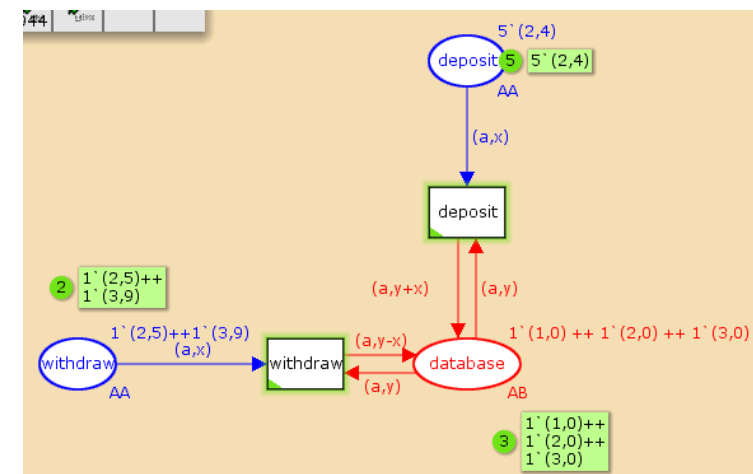
main'withdraw 1 $1^{\sim}(2,5)++1^{\sim}(3,9)$

Best Lower Multi-set Bounds

main'database 1 $1^{\sim}(1,0)$

main'deposit 1 empty

main'withdraw 1 empty



Report (3)

Home Markings
[24]

Liveness Properties

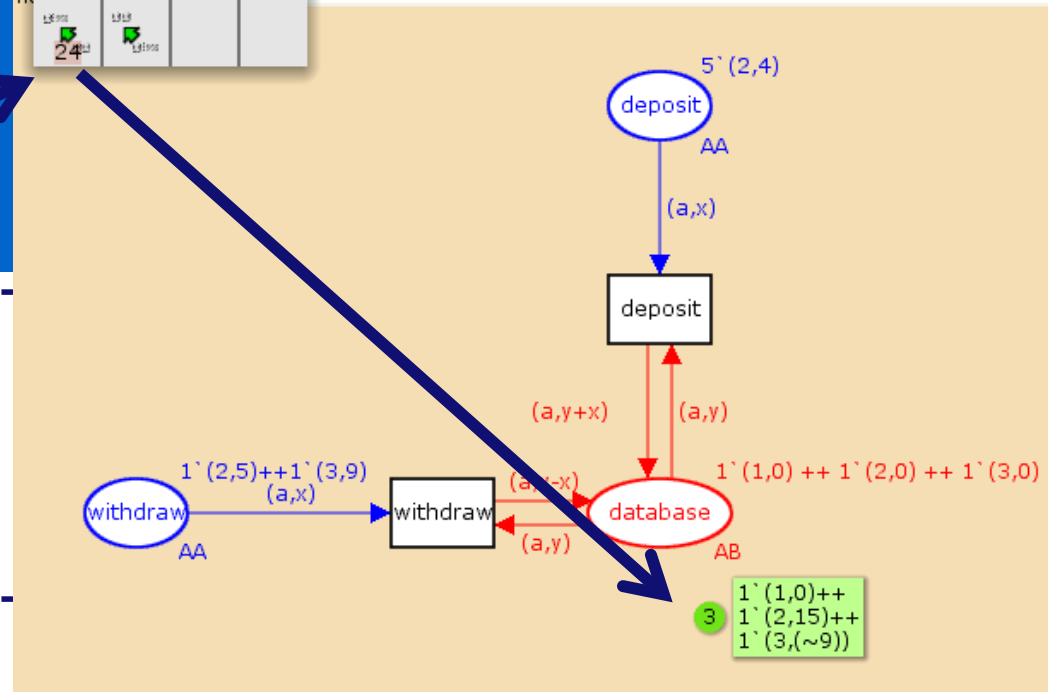
Dead Markings
[24]

Dead Transition Instances
None

Live Transition Instances
None

Fairness Properties

No infinite occurrence sequences.



- ▶ Tool box
- ▶ Help
- ▶ Options
- ▼ train_folded.cpn

Step: 0
Time: 0

- ▶ Options
- ▶ History
- ▼ Declarations

▶ Standard declarations

```

▼ colset Track = int with 0..6;
▼ colset TState = with busy | free;
▼ colset TS = product Track*TState;
▼ colset Train = with A|B;
▼ colset TT = product Train * Track;
▼ var tk,tk1,tk2:Track;
▼ var tn,tn1,tn2:Train;

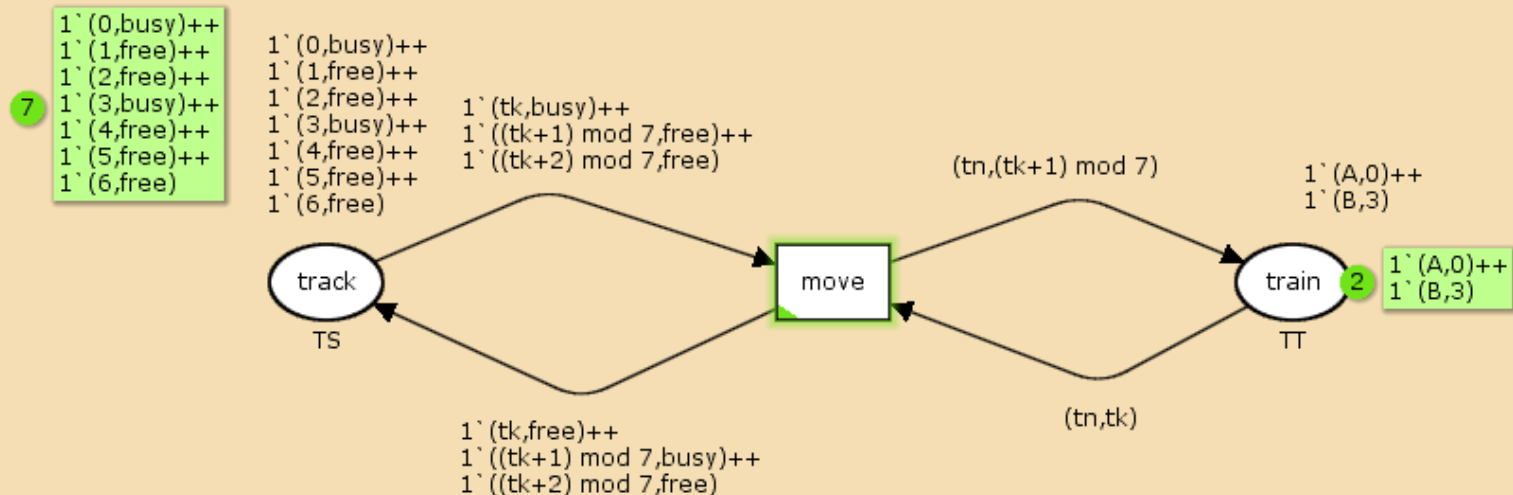
```

- ▶ Monitors

[New Page](#)

Binder 0

[New Page](#)



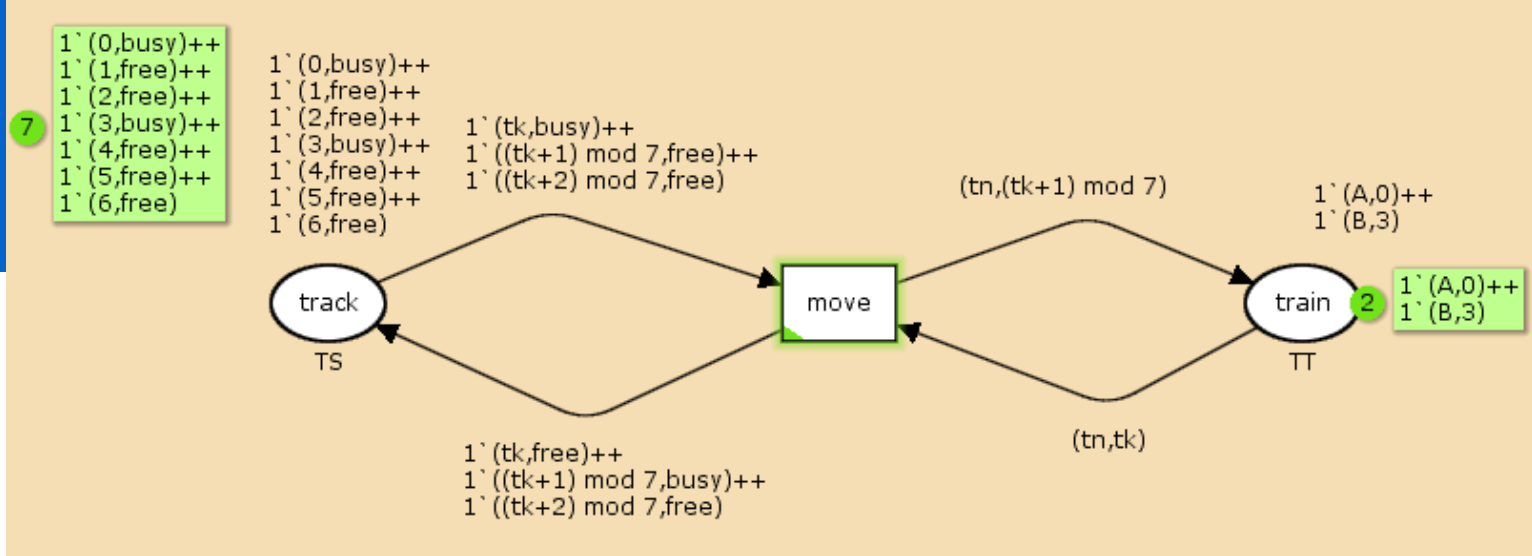
State Space
Nodes: 28
Arcs: 42
Secs: 0
Status: Full

Scc Graph
Nodes: 1
Arcs: 0
Secs: 0

Boundedness Properties

Best Integer Bounds

	Upper	Lower
New_Page'track 1	7	7
New_Page'train 1	2	2



Home Properties

Home Markings
All

Liveness Properties

Dead Markings
None

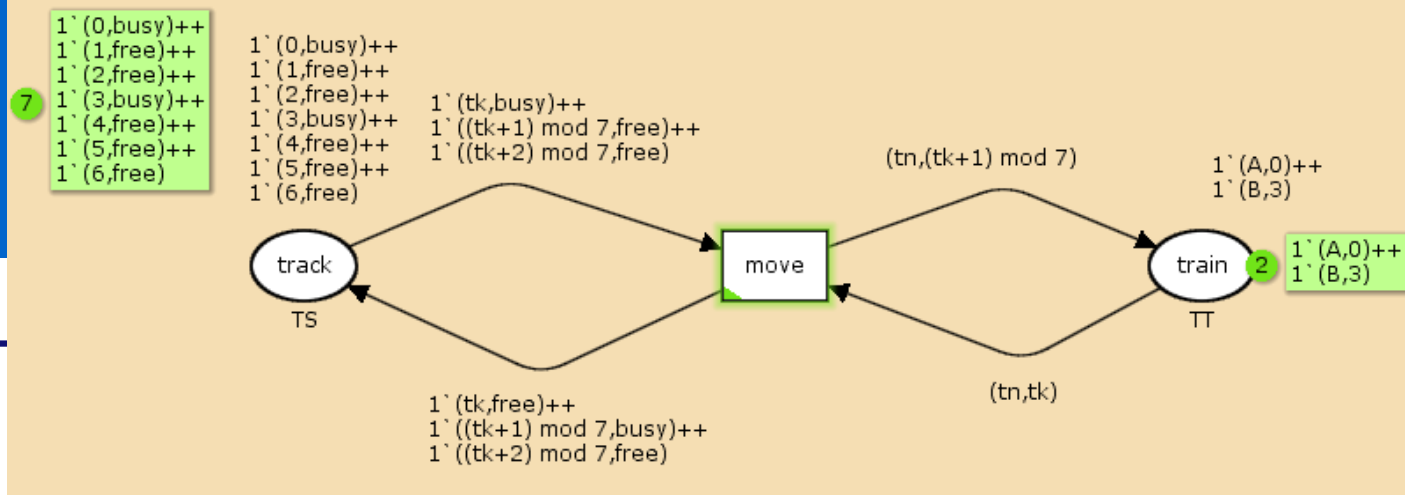
Dead Transition Instances
None

Live Transition Instances
All

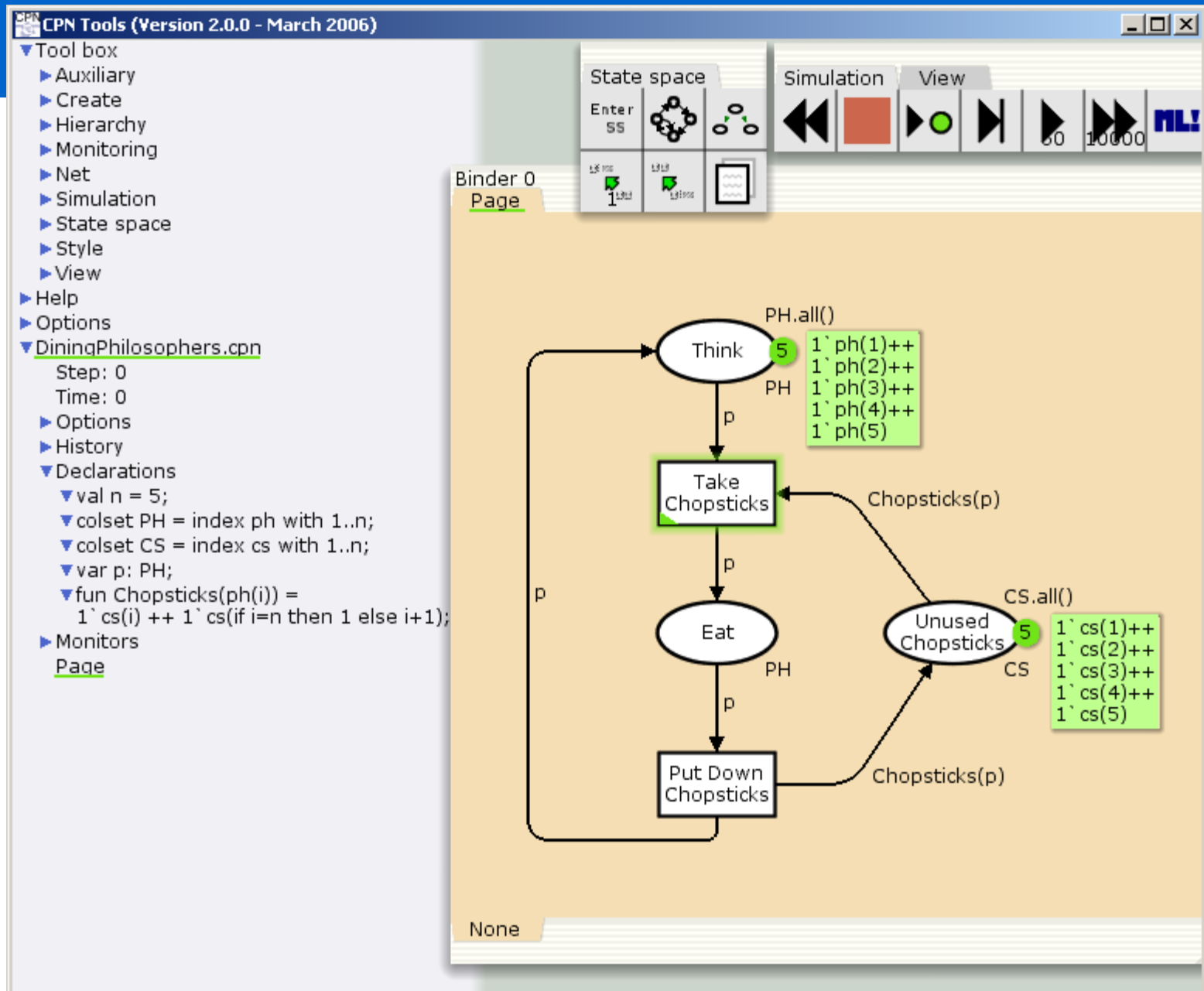
Fairness Properties

New_Page'move 1

Impartial



Another example



Report (1)

CPN Tools state space report for:

C:\Program Files\CPN Tools\Samples\DiningPhilosophers\DiningPhilosophers.cpn

Report generated: Thu Nov 02 10:42:53 2006

Statistics

State Space

Nodes: 11

Arcs: 30

Secs: 0

Status: Full

Scc Graph

Nodes: 1

Arcs: 0

Secs: 0

Report (2)

Boundedness Properties

Best Integer Bounds

Page'Eat 1
Page'Think 1
Page'Unused_Chopsticks 1

Upper	Lower
2	0
5	3
5	1

Best Upper Multi-set Bounds

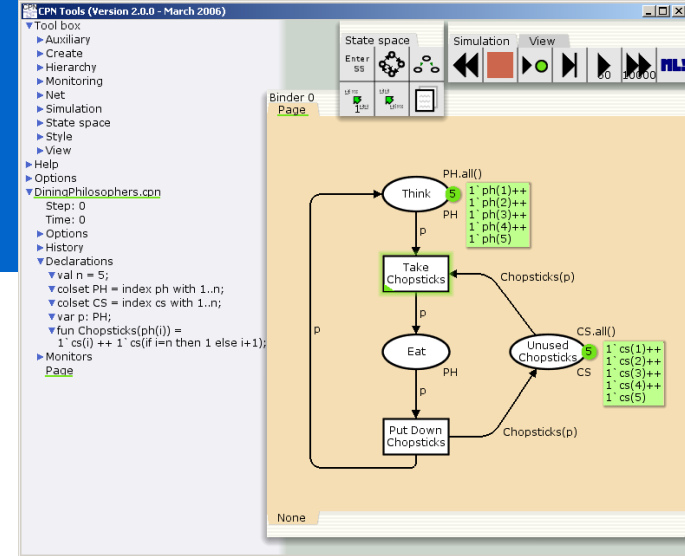
Page'Eat 1
1`ph(5)
Page'Think 1
1`ph(5)
Page'Unused_Chopsticks 1

1`ph(1)++ 1`ph(2)++ 1`ph(3)++ 1`ph(4)++
1`ph(1)++ 1`ph(2)++ 1`ph(3)++ 1`ph(4)++
1`cs(1)++ 1`cs(2)++ 1`cs(3)++ 1`cs(4)++ 1`cs(5)

Best Lower Multi-set Bounds

Page'Eat 1
Page'Think 1
Page'Unused_Chopsticks 1

empty
empty
empty



Report (3)

Home Properties

Home Markings
All

Liveness Properties

Dead Markings
None

Dead Transition Instances
None

Live Transition Instances
All

Fairness Properties

Page'Put_Down_Chopsticks 1
Page'Take_Chopsticks 1

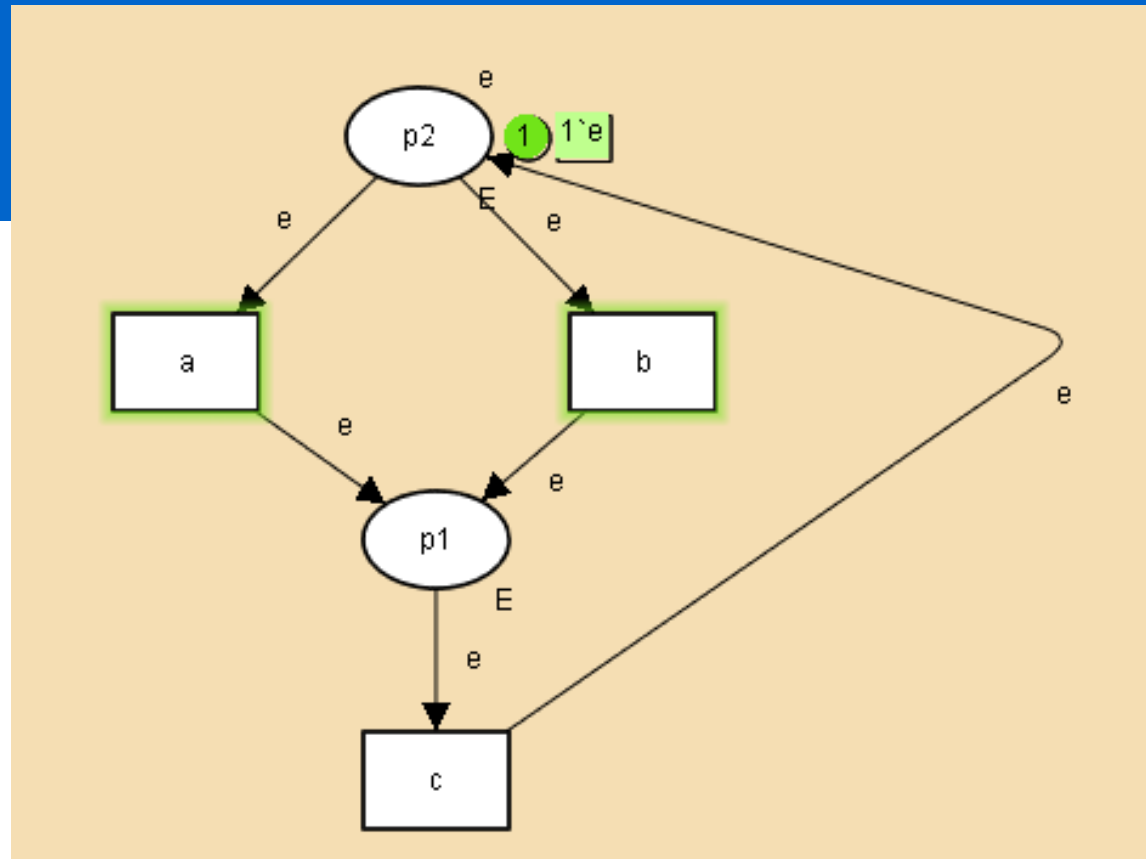
Impartial
Impartial

strongest fairness property, i.e., there are infinite firing sequences and in each infinite firing sequence t occurs infinitely often

Fairness properties

- Are only relevant if there are Infinite Firing Sequences (IFS), otherwise CPN Tools reports: "no infinite occurrence sequences".
- Given a transition t it is often desirable that t appears infinitely often in an IFS.
- Properties reported by CPN Tools
 - t is **impartial**: t occurs infinitely often in every IFS.
 - t is **fair**: t occurs infinitely often in every IFS where t is enabled infinitely often.
 - t is **just**: t occurs infinitely often in every IFS where t is continuously enabled from some point onward
 - **No fairness**: not just, i.e., there is an IFS where t is continuously enabled from some point onward and does not fire anymore

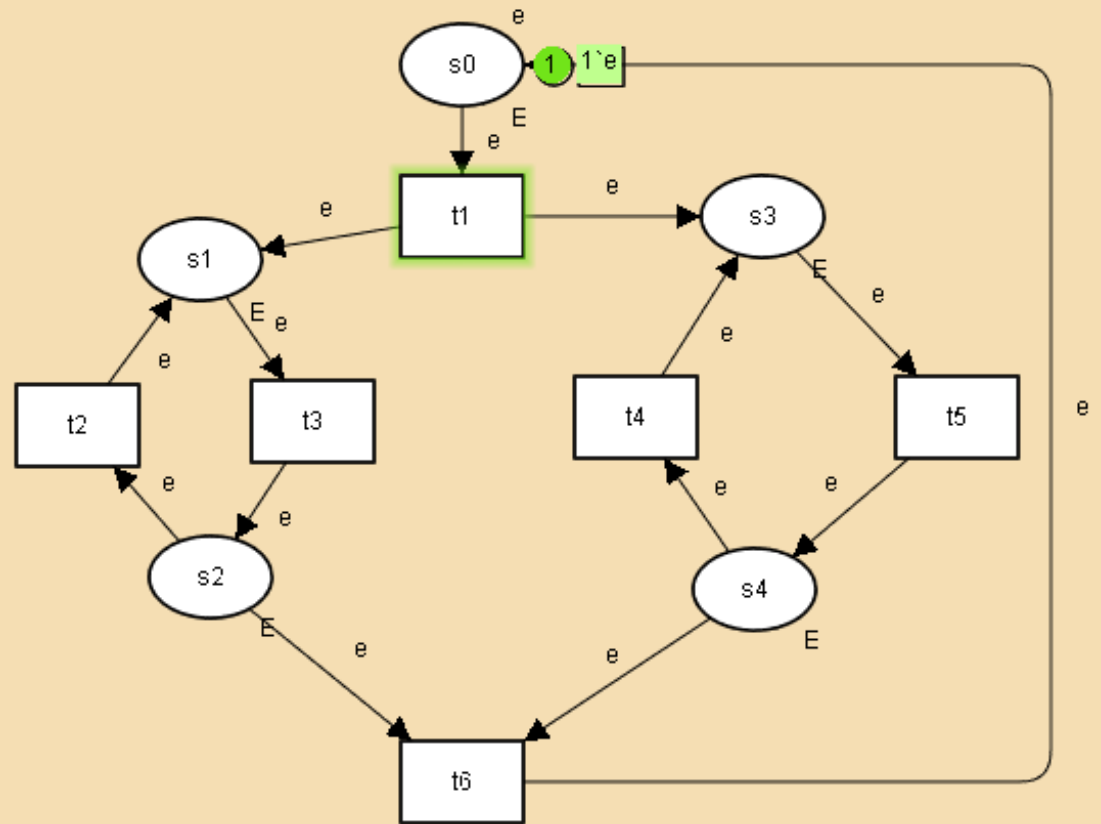
Example



Fairness Properties

main1'a 1	Just
main1'b 1	Just
main1'c 1	Impartial

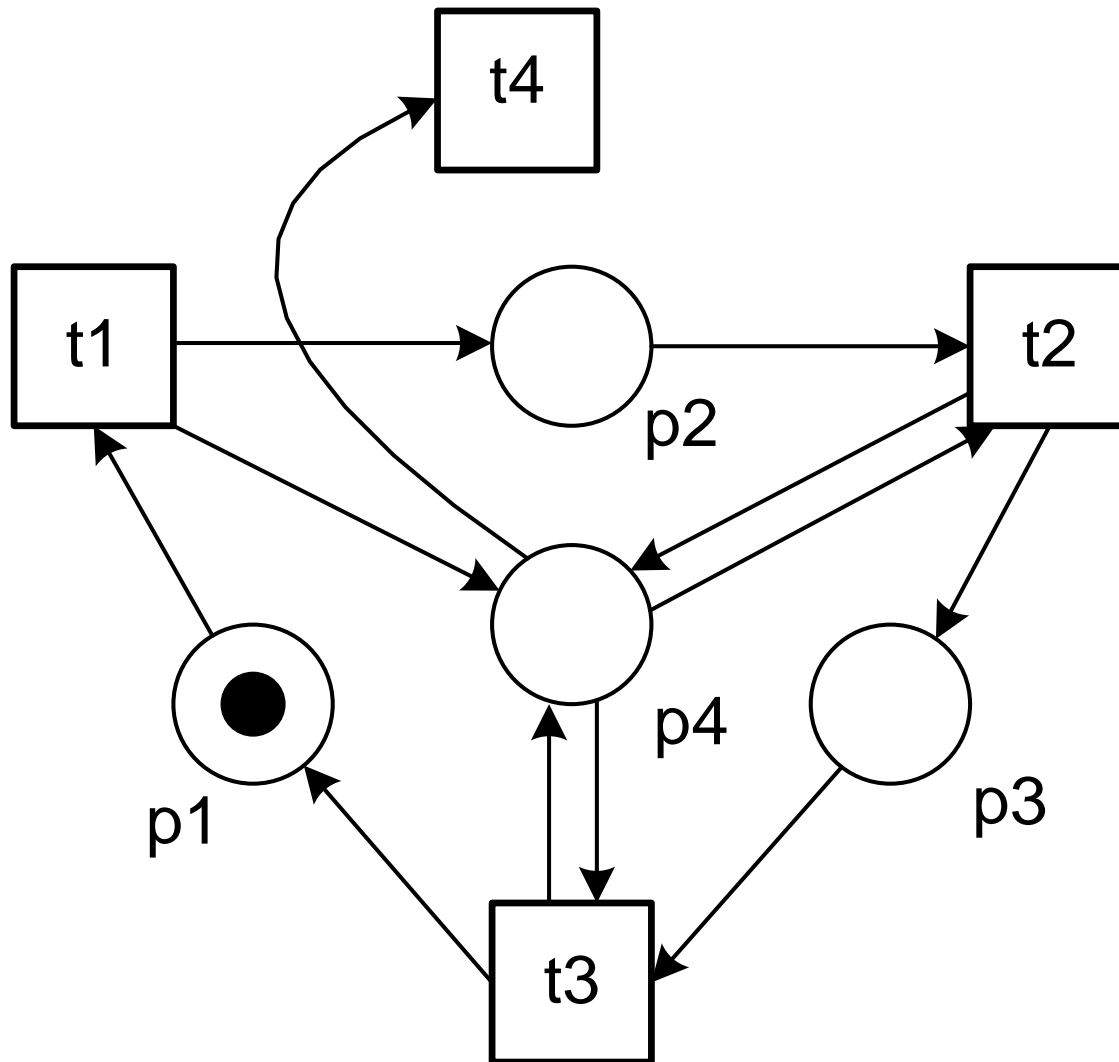
Example



Fairness Properties

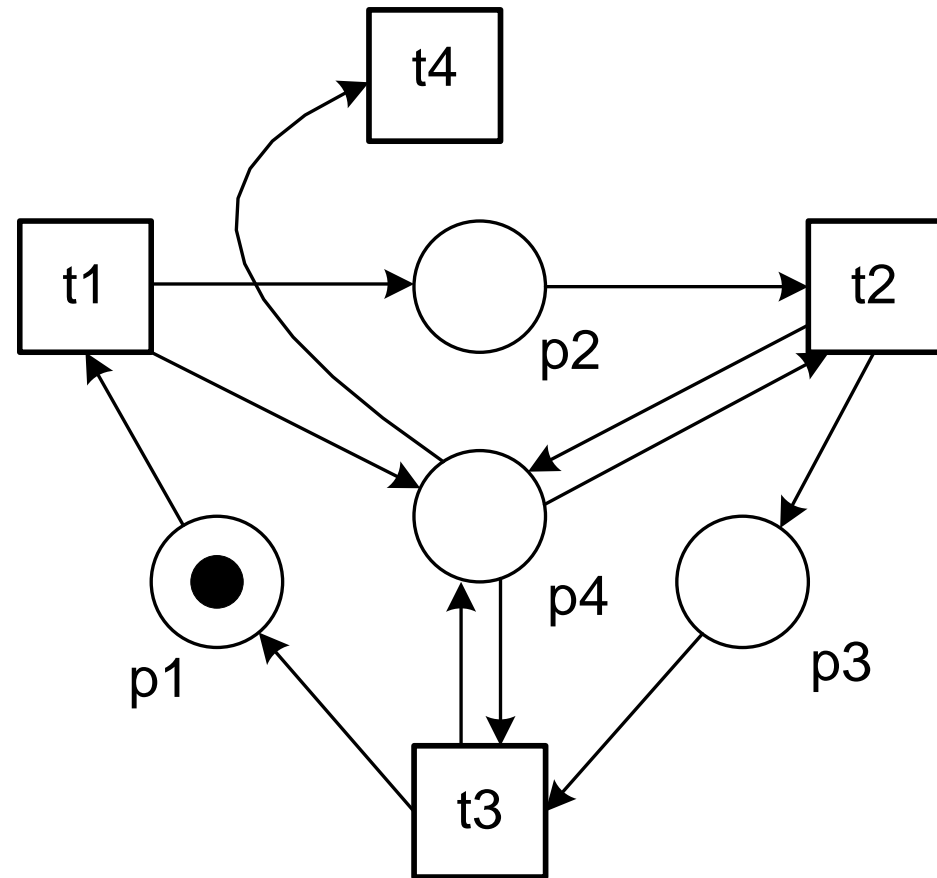
main3't1 1	Fair
main3't2 1	No Fairness
main3't3 1	No Fairness
main3't4 1	No Fairness
main3't5 1	No Fairness
main3't6 1	Just

Exercise



Indicate for each transition whether it is *impartial*, *fair*, or *just* (or satisfies *no fairness property*)

- **t1, t2, and t3 are all impartial because it is not possible to construct an infinite firing sequence where not all of these transitions appear infinitely often. If one stops executing one of these transitions, the system will block after a while.**
- **t4 has no fairness as it is possible to construct an infinite firing sequence where t4 remains enabled but never fires.**



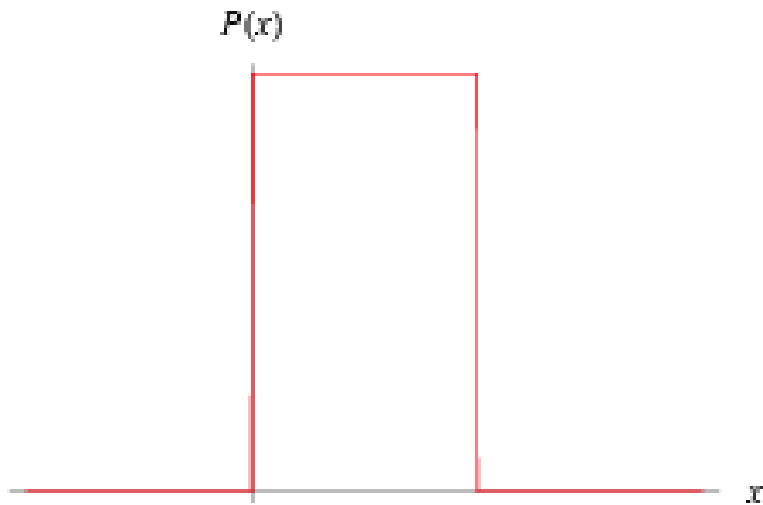
Simulation

- **Most widely used analysis technique.**
- **From a technical point of view just a "walk" in the reachability graph.**
- **By making many "walks" (in case of transient behavior) or a very "long walk" (in case of steady-state) behavior, it is possible to make reliable statements about properties/ performance indicators.**
- **Used for validation and performance analysis.**
- **Cannot be used to prove correctness!**

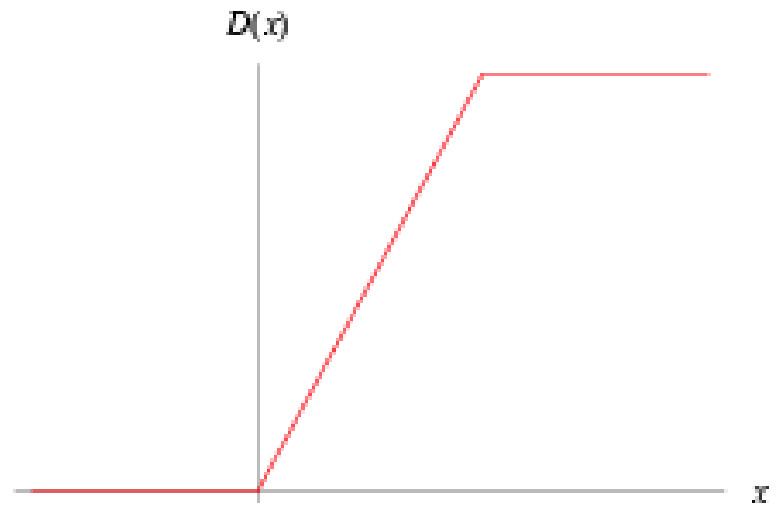
Stochastic process

- Simulation of a deterministic system is not very interesting.
- Simulation of an untimed system is not interesting.
- In a timed and non-deterministic system, durations and probabilities are described by some **probability distribution**.
- In other words, we simulate a stochastic process!
- CPN allows for the use of distributions using some internal random generator.

Uniform distribution

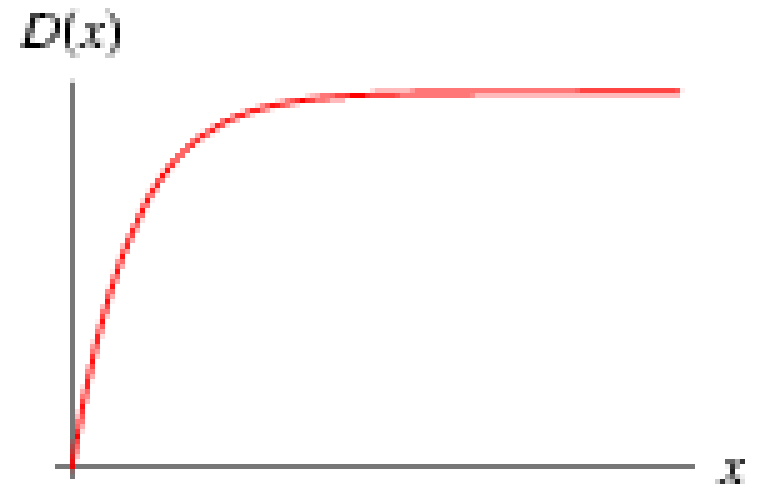
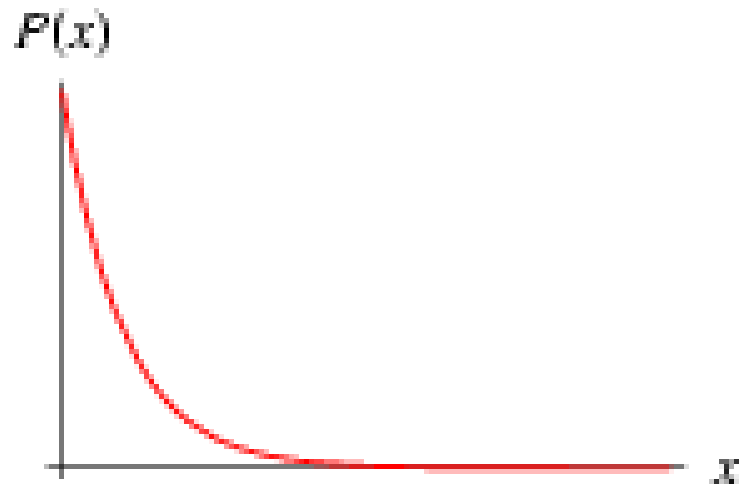


probability density function
(PDF)

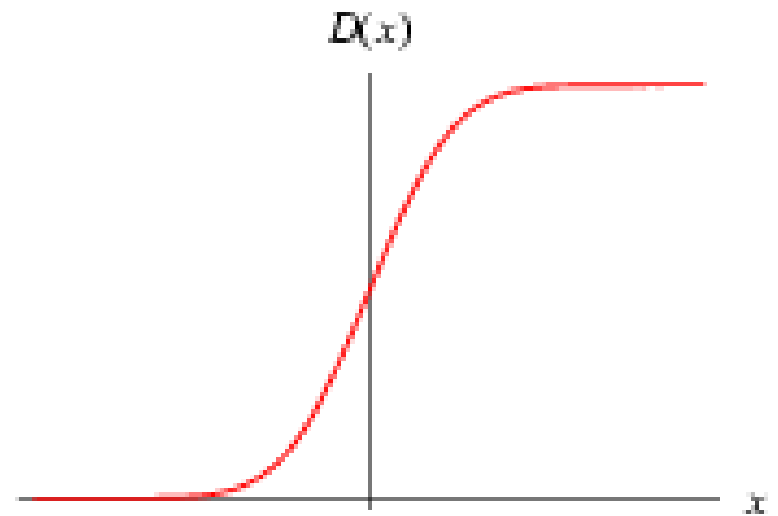
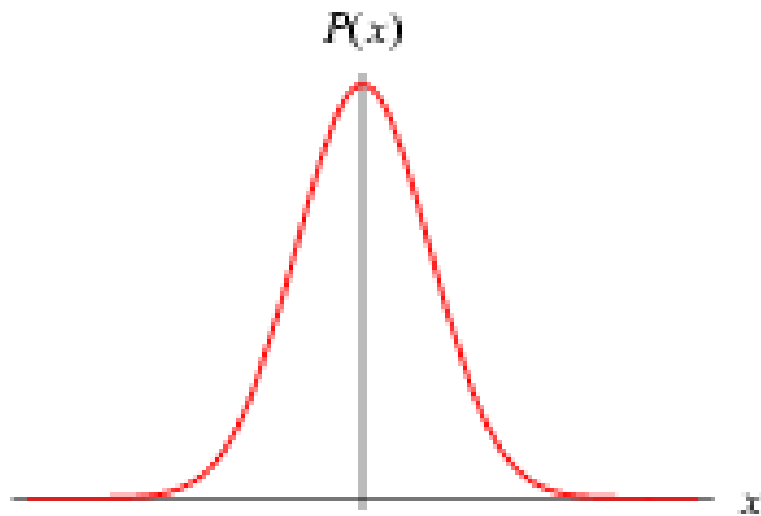


cumulative distribution function
(CDF)

Negative exponential distribution



Normal distribution



Distributions in CPN Tools

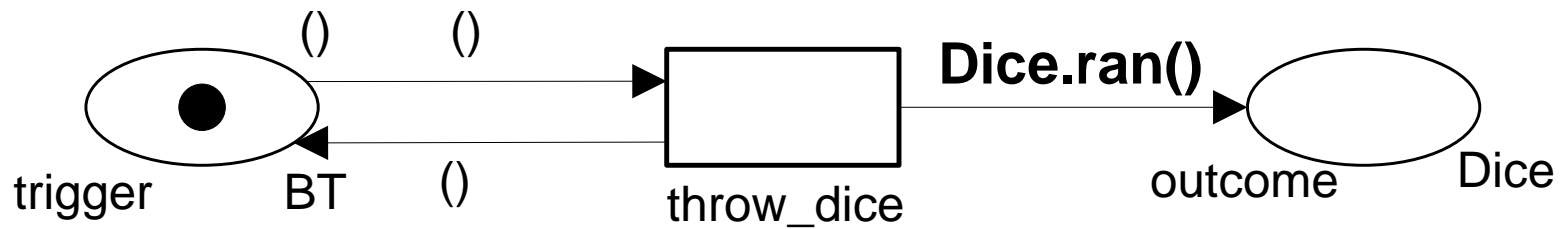
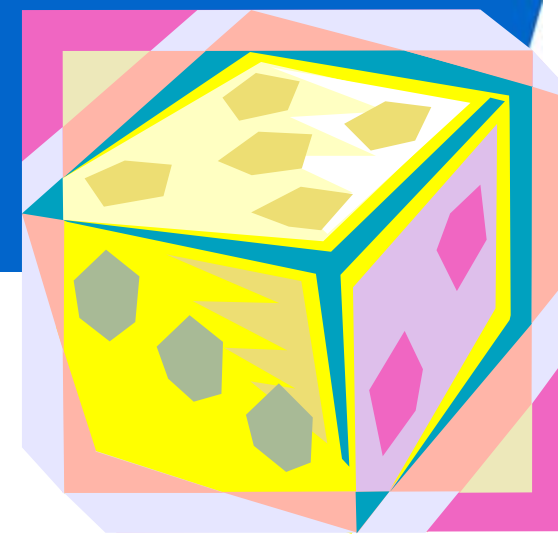
There is standard library with stochastic functions:

- `uniform(a:real, b:real) : real`
- `exponential(r:real) : real`
- `normal(n:real, v:real) : real`
- `erlang (n:int, r:real) : real`
- Etc.

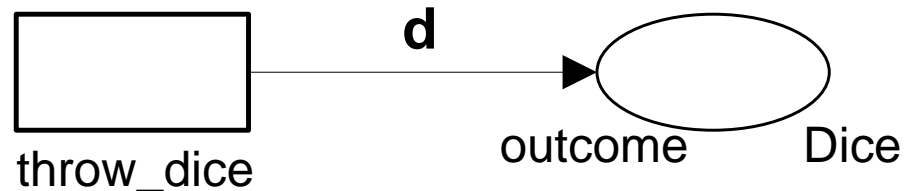
A nice additional function is also `C.ran()` which returns a randomly selected element of finite color set `C`, e.g.,
color C = int with 1..5;
fun select1to5() = C.ran()
returns a number between 1 and 5

Example

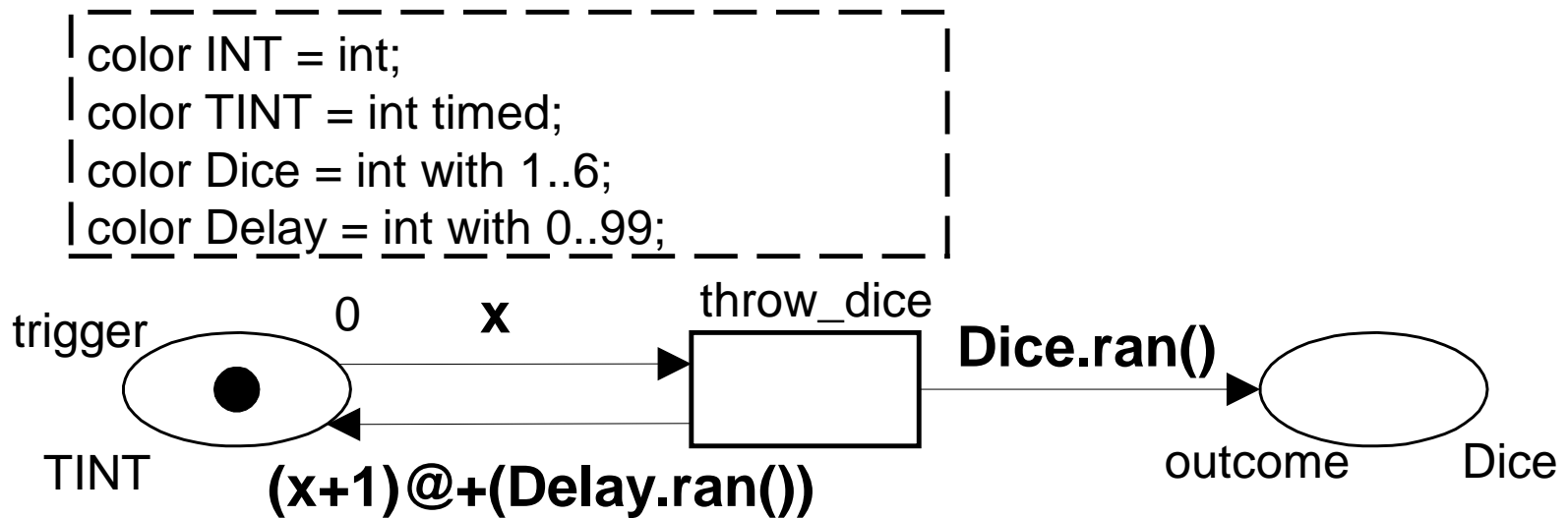
```
color BT = unit;  
color Dice = int with 1..6;  
var d : Dice;
```



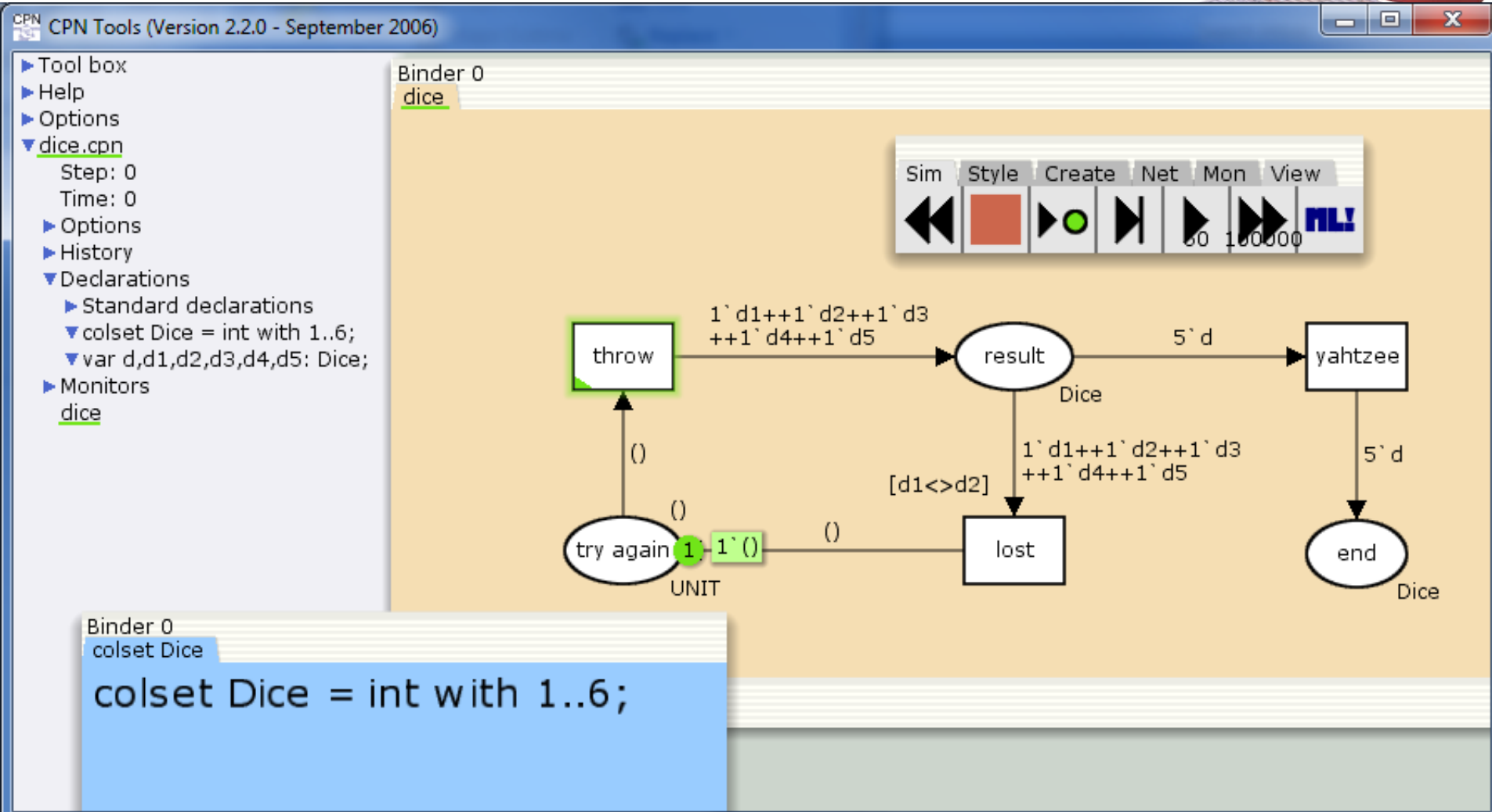
or even simpler ...



Example(2)

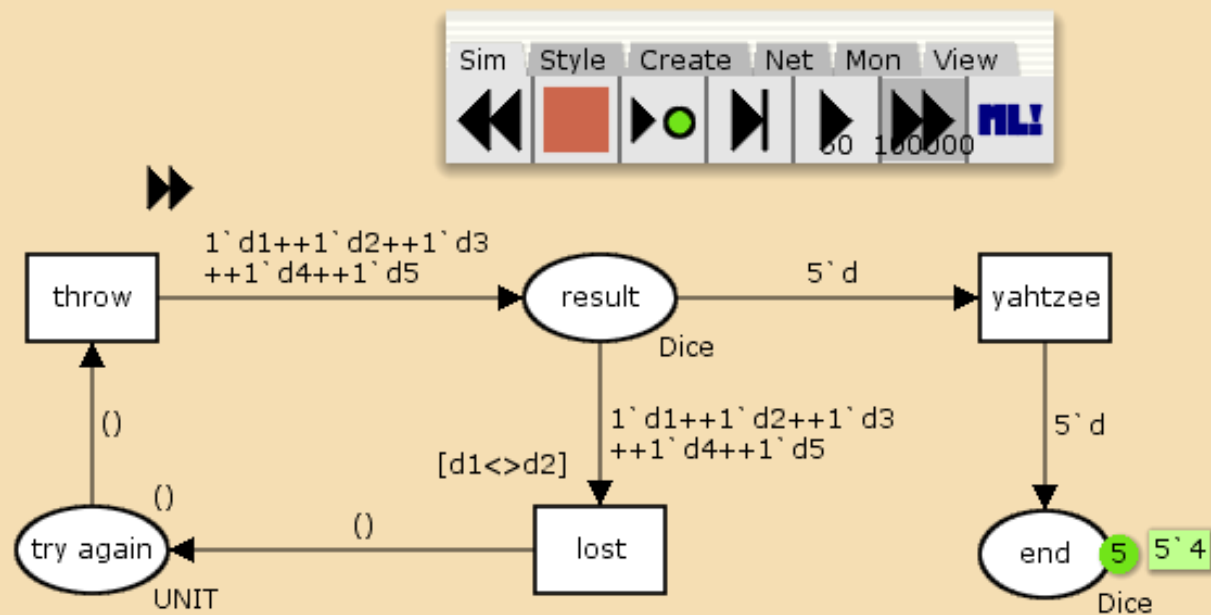


A red plastic Yahtzee game set is shown. The base is a red circular tray with a green interior. The lid is also red and features the word "YAHITZEE!" in white, stylized letters. The lid is propped open, revealing the green interior. Five white dice are placed on the rim of the base. A red plastic cup is visible in the foreground, and a portion of a Yahtzee scorecard is visible at the bottom.



After 2055 times throwing the dices ... five 4's

▼ dice.cpn
Step: 4110
Time: 0
► Options
► History
▼ Declarations
► Standard declarations
▼ colset Dice = int with 1..6;
▼ var d,d1,d2,d3,d4,d5: Dice;
► Monitors
dice



Binder 0
colset Dice
colset Dice = int with 1..6;

- ▶ Tool box
- ▶ Help
- ▶ Options
- ▼ statistics.cpn

Step: 0
Time: 0

- ▶ Options
- ▶ History
- ▼ Declarations
 - ▶ Standard declarations
 - ▼ colset Job = int timed;
 - ▼ colset Res = unit;
 - ▼ var j:Job;
 - ▼ var r:Res;
 - ▼ fun iat() = round(exponential(0.05));
 - ▼ fun pt() = round(normal(10.0,1.0));
 - ▼ var result:INT;
- ▶ Monitors
 - example functions
 - example net

Binder 0

example functions example net



Hier



bernoulli(0.5) val it = 0 : int

binomial(10,0.5) val it = 5 : int

discrete (1,10) val it = 6 : int

erlang(10,0.5) val it = 23.9142997372 : real

exponential(100.0) val it = 0.0112478388921 : real

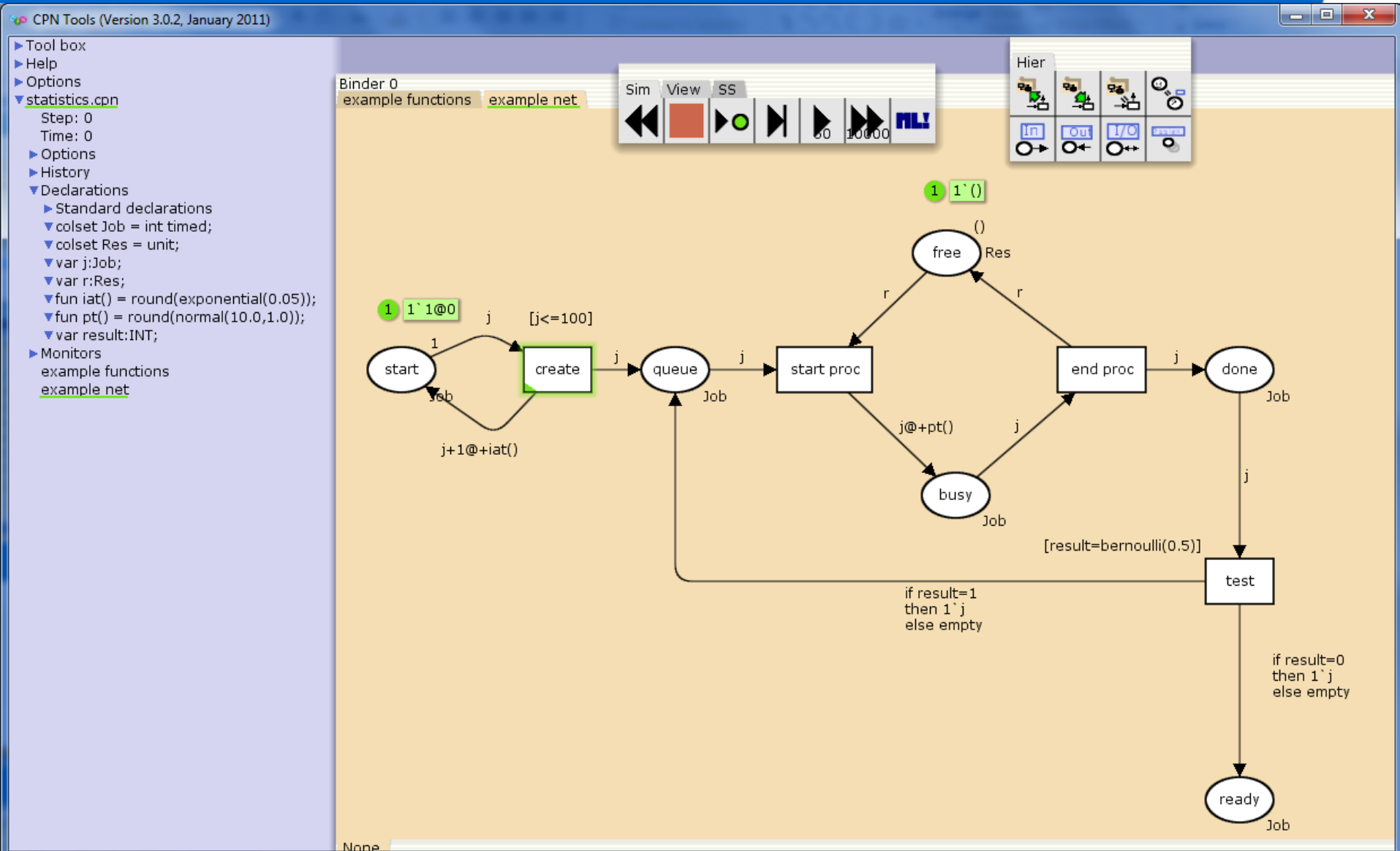
normal(10.0,5.0) val it = 8.85471291714 : real

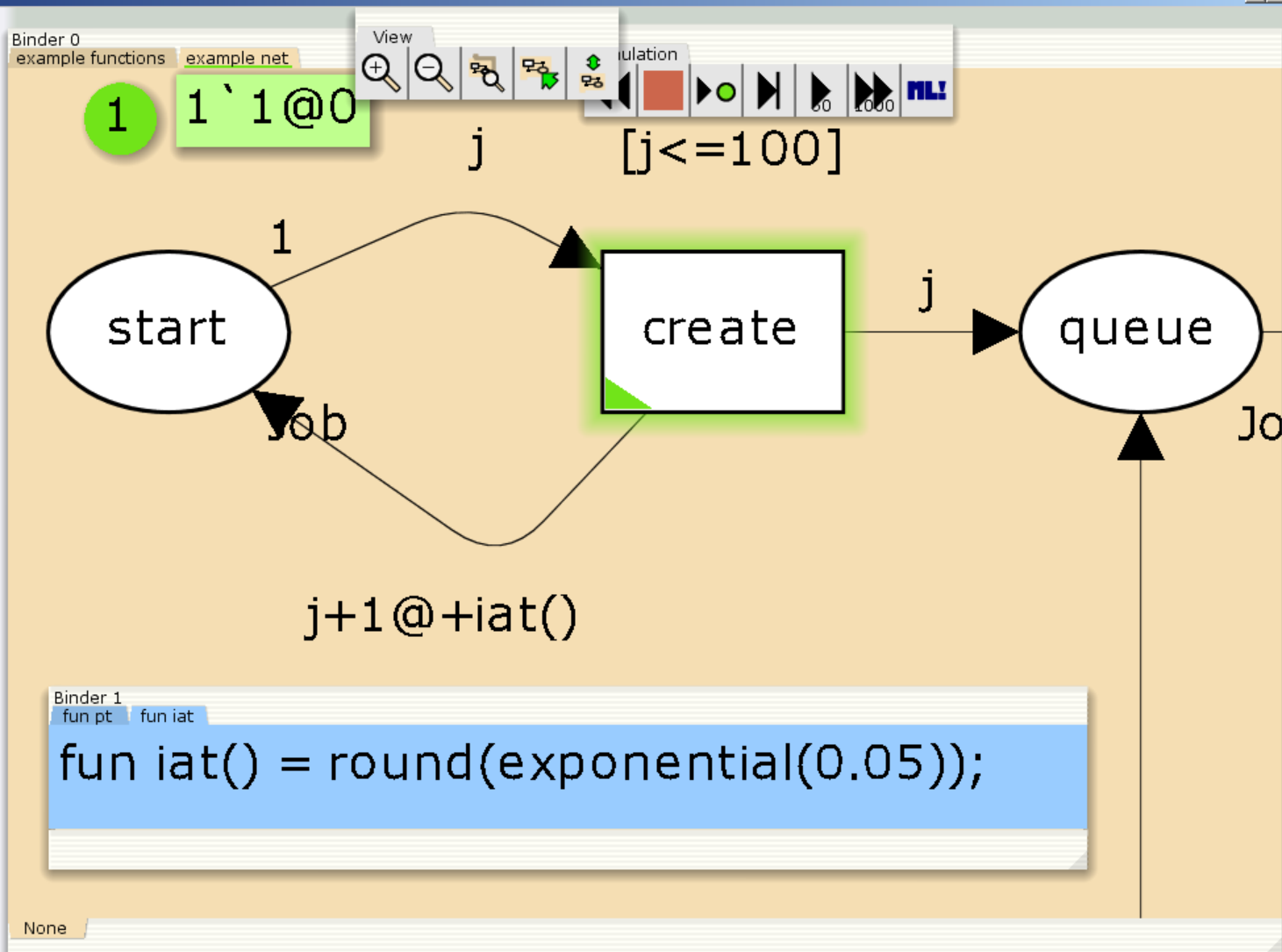
poisson(0.5) val it = 1 : int

uniform(10.0,20.0) val it = 16.3058542628 : real

None

Example





- ▼ Tool box
 - ▶ Auxiliary
 - ▶ Create
 - ▶ Hierarchy
 - ▶ Monitoring
 - ▶ Net
 - ▶ Simulation
 - ▶ State space
 - ▶ Style
 - ▶ View
 - ▶ Help
 - ▶ Options
- ▼ statistics.cpn
 - Step: 0
 - Time: 0
 - ▶ Options
 - ▶ History
- ▼ Declarations
 - ▼ Standard declarations
 - ▶ colset UNIT
 - ▶ colset INT
 - ▼ colset BOOL = bool;
 - ▶ colset STRING
 - ▼ colset C = int with 1..5;
 - ▼ colset Job = int timed;
 - ▼ colset Res = unit;
 - ▼ var j:Job;
 - ▼ var r:Res;
 - ▼ fun iat() = round(exponential(0.0));
 - ▼ fun pt() = round(normal(10.0,1.0));
 - ▼ var result:INT;
- ▶ Monitors
 - example functions
 - example net

Binder 0
example functions example net

View



Simulation



j

start proc

end

j@+pt()

j

busy

Job

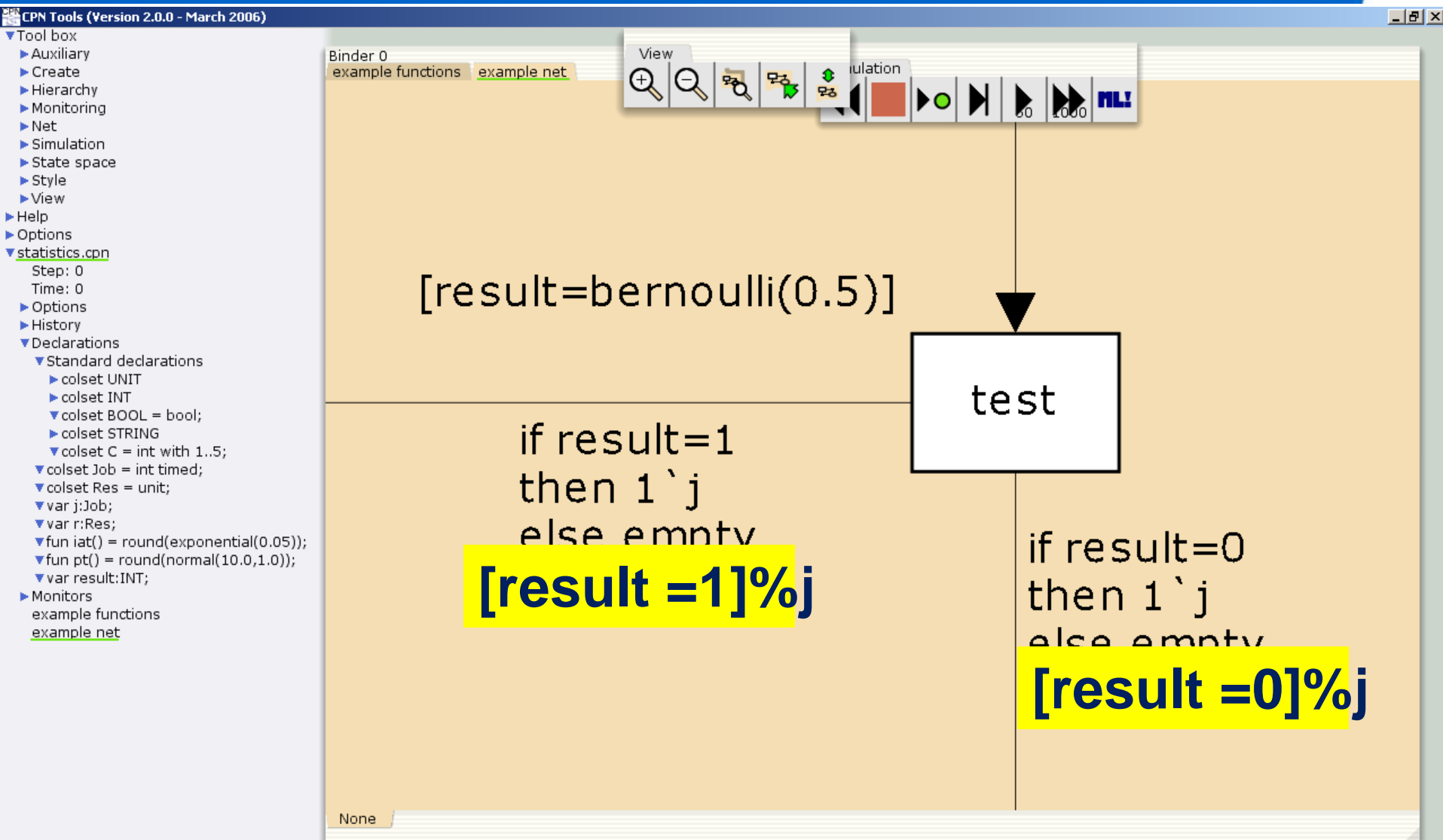
[result

Binder 1
fun pt

```
fun pt() = round(normal(10.0,1.0));
```

None

alternative notation $[b]\%v = \text{if } b \text{ then } 1`v \text{ else empty}$





Example revisited

CPN Tools (Version 2.0.0 - March 2006)

- Tool box
- Help
- Options

ct_smart_stochastic.cpn

Step: 300

Time: 368

- Options

- History

- Declarations

- Standard declarations

- colset E
- colset INT
- colset BOOL
- colset STRING
- colset Prod
- colset PL
- colset Res
- colset PR
- var p sel
- var r
- var l
- var i

```

fun delay(p,r) =
  round(exponential(1.0/real(
    if p="tea" then
      (if r="Eve" then 2 else 6) else
      (if r="Eve" then 12 else 4)
    )))

```

```

fun interarrivaltime(p)=
  round(exponential(1.0/real(
    if p = "tea" then 5 else 10
    ))) ;

```

```

fun select(p,l,r) =
  if (p="tea" andalso r="Eve") orelse
  (p="coffee" andalso r="Adam") orelse
  l = []
  then p
  else select(hd(l),tl(l),r)

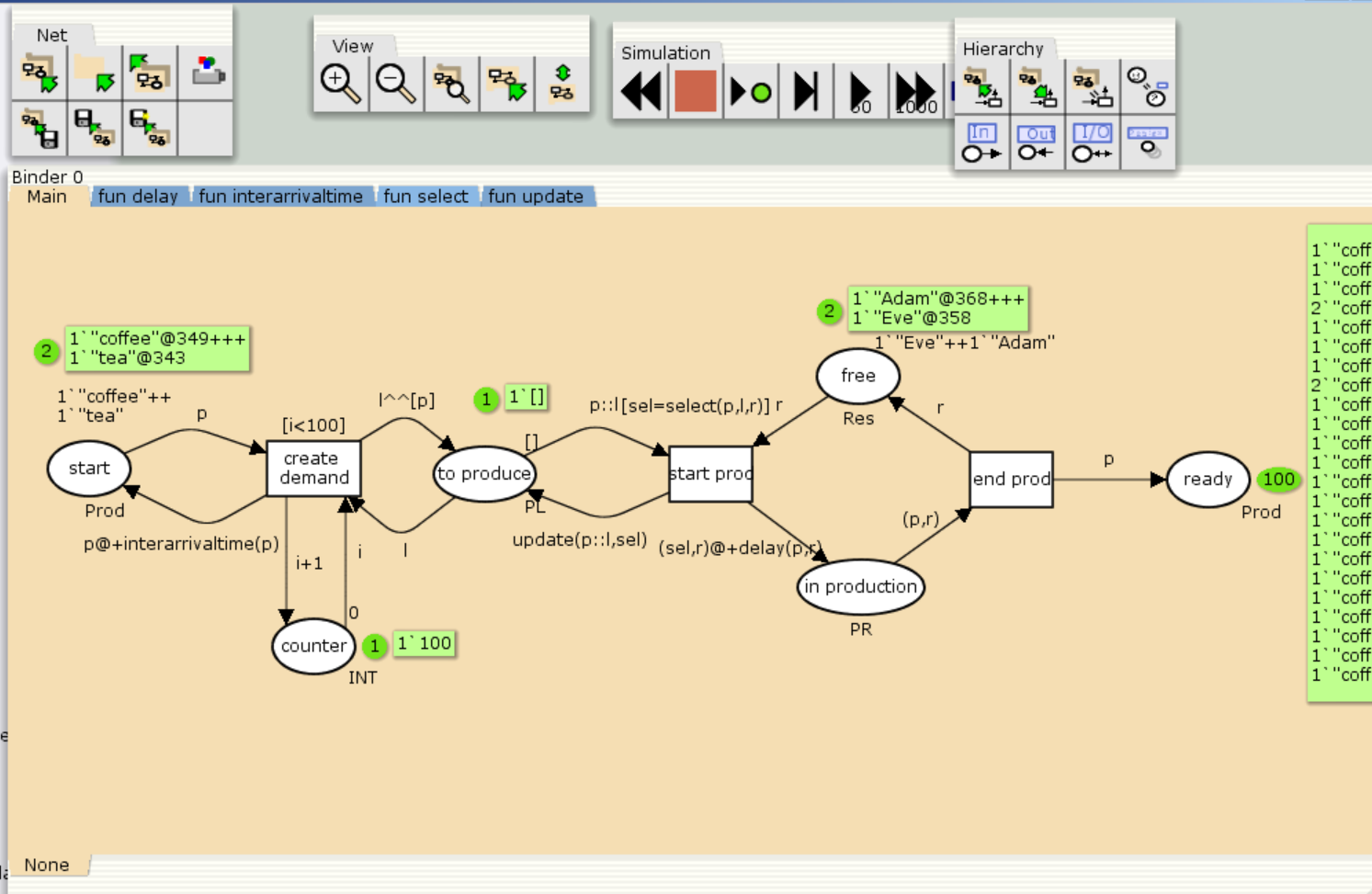
```

```

fun update(l,p) =
  if l = []
  then []
  else if hd(l) = p then tl(l) else p::update

```

- Monitors
- Main

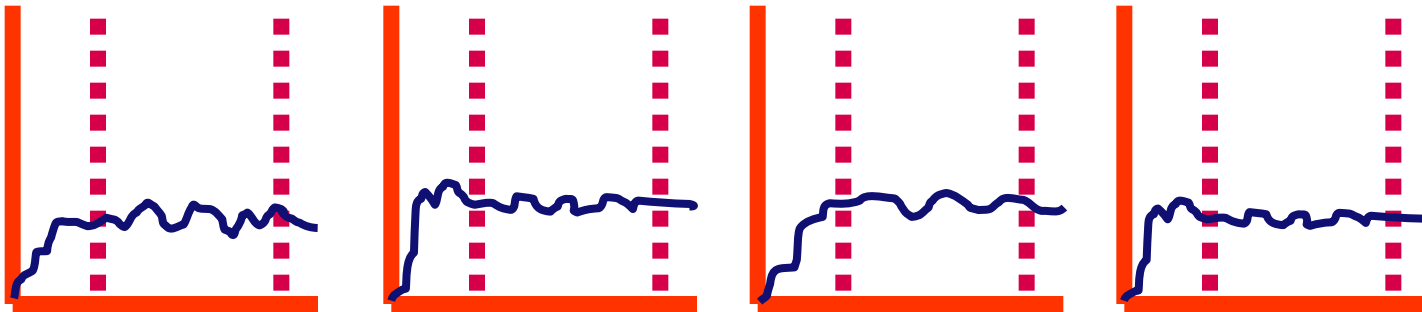


Subruns and confidence intervals

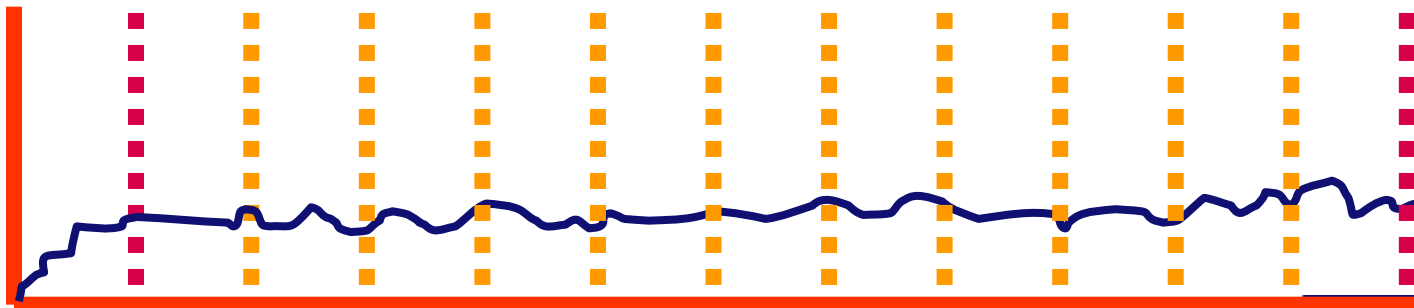
- A single run does **not** provide information about reliability of results.
- Therefore, multiple runs or one run cut into parts: **subruns**.
- If the subruns are assumed to be **mutually independent**, one can calculate a **confidence interval**, e.g., the flow time is with 95% confidence within the interval 5.5 ± 0.5 (i.e. [5,6]).

Two possible settings

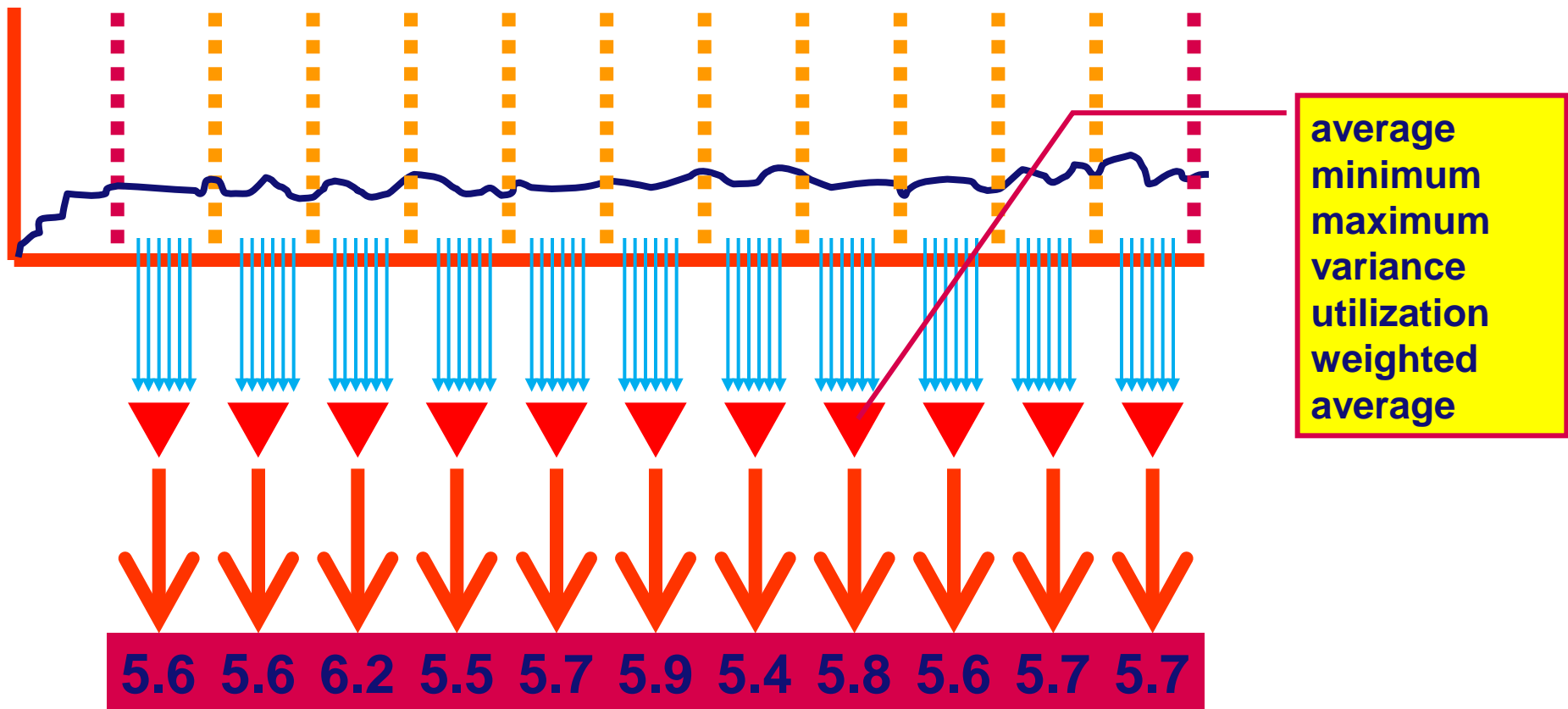
Steady-state analysis (I)



Steady-state analysis (II)



More on calculating confidence intervals

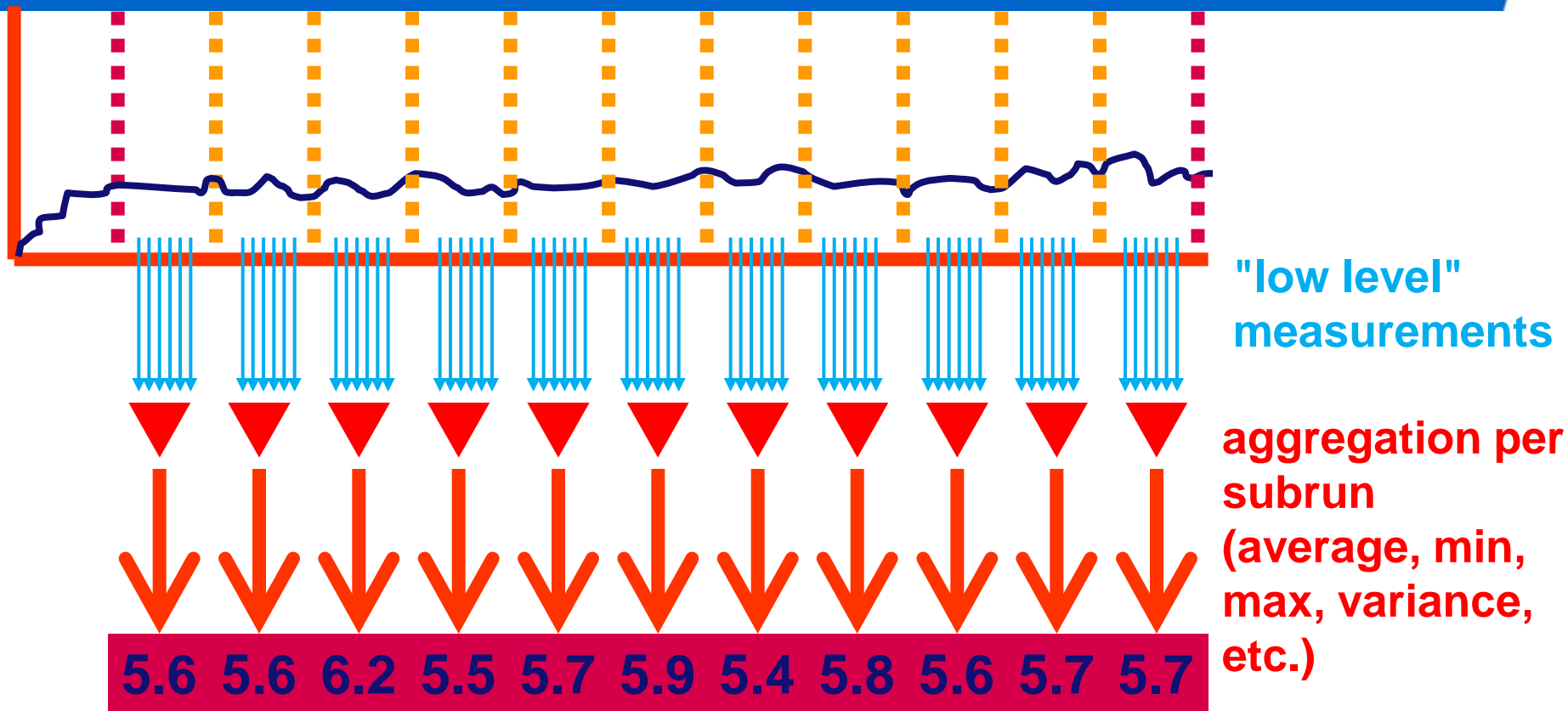


5.6 5.6 6.2 5.5 5.7 5.9 5.4 5.8 5.6 5.7 5.7

is not the same as

4.6 6.6 3.2 8.5 1.7 9.9 4.4 6.8 4.6 6.7 5.7

although the average over the
subrun results is the same (5.7)

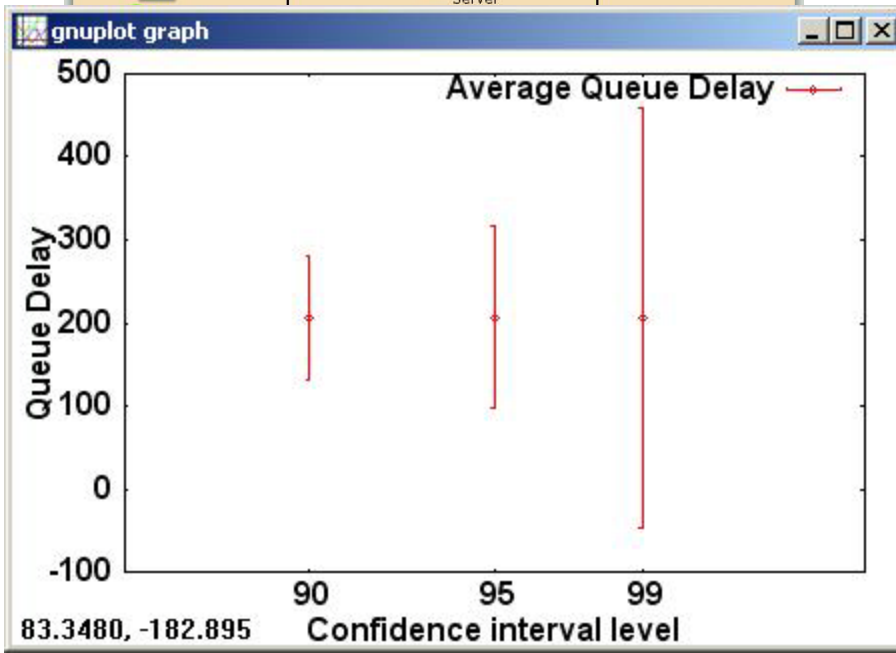
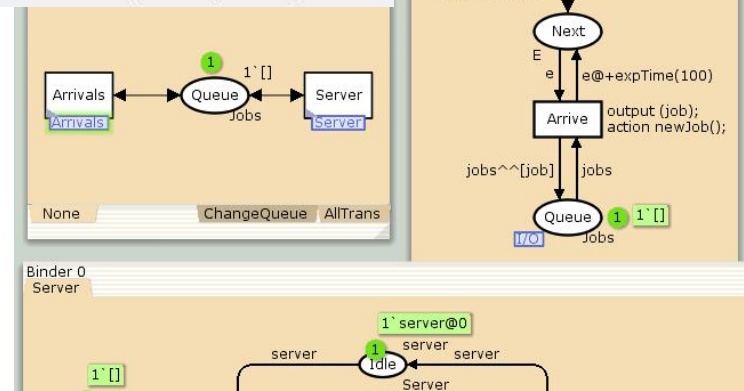


subruns = 11
average = 5.7
standard deviation = 0.21

confidence = 0.9
confidence interval =
 $[5.7 - 0.117, 5.7 + 0.117] = [5.58, 5.82]$

Using monitors in CPN Tools

- ▼ Marking_size_Top'A_1
 - ▼ Type: Marking size
 - Logging
 - ▼ Nodes ordered by pages
 - ▼ Top
- Top'A 1 (Place)



CPN Tools Simulation Performance Report
 Net: C:\nets\QueueSystem\QueueSystem.cpn

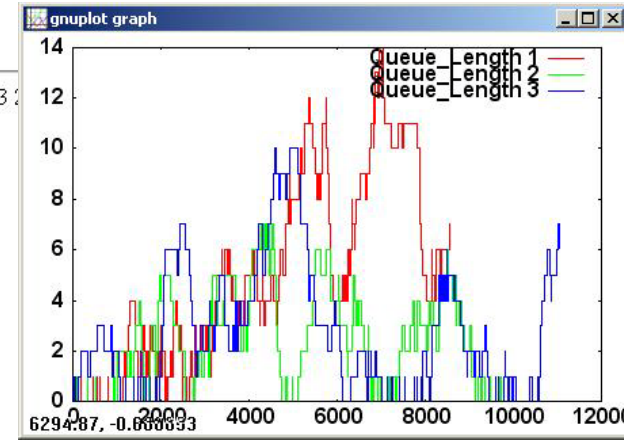
Note that these statistics have been calculated for data that is not necessarily independent or identically distributed.

Timed statistics					
Name	Count	Avg	Min	Max	Time Interval
Marking_size_Server'Busy_1	201	0.828407	0	1	11131
Queue_Length	209	1.992993	0	9	11131
Server_Utilization	118	0.828407	0	1	11131

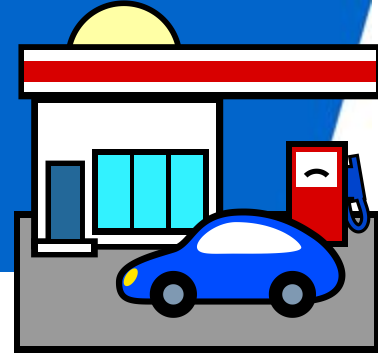
Untimed statistics						
Name	Count	Sum	Avg	StD	Min	Max
Count_trans_occur_Arrivals'Arrive_1	107	107	1.000000	0.000000	1	1
Processed_A_Jobs	46	46	1.000000	0.000000	1	1
Queue_Delay	100	19933	199.330000	252.527148	0	1255

Simulation steps executed: 307
 Model time: 11131

Generated: Wed Feb 15 15:11:03



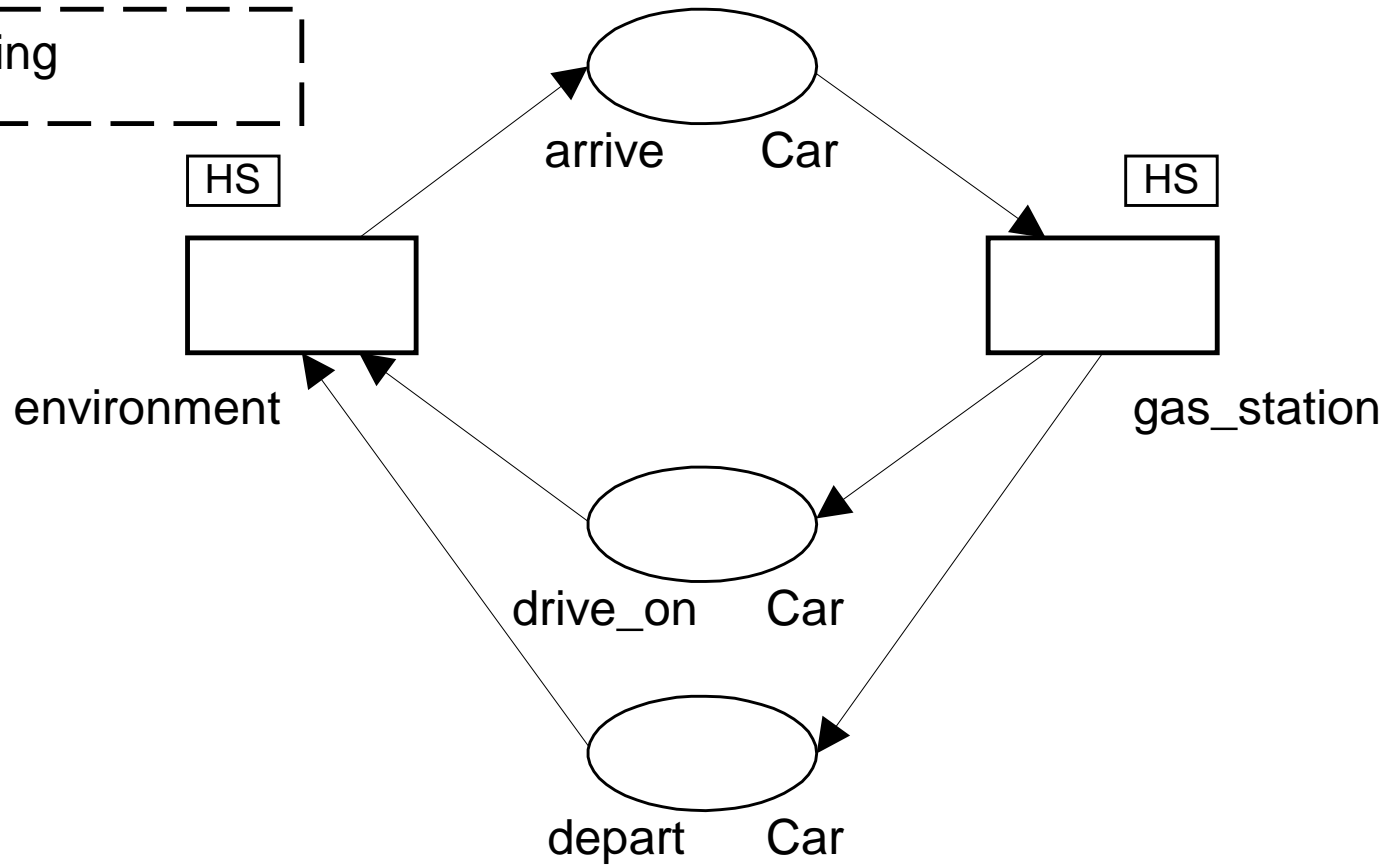
Example of a simulation model



- Gas station with one pump and space for 4 cars (3 waiting and 1 being served).
- Service time: uniform distribution between 2 and 5 minutes.
- Poisson arrival process with mean time between arrivals of 4 minutes.
- If there are more than 3 cars waiting, the "sale" is lost.
- Questions: flow time, waiting time, utilization, lost sales, etc.

Top-level page: *main*

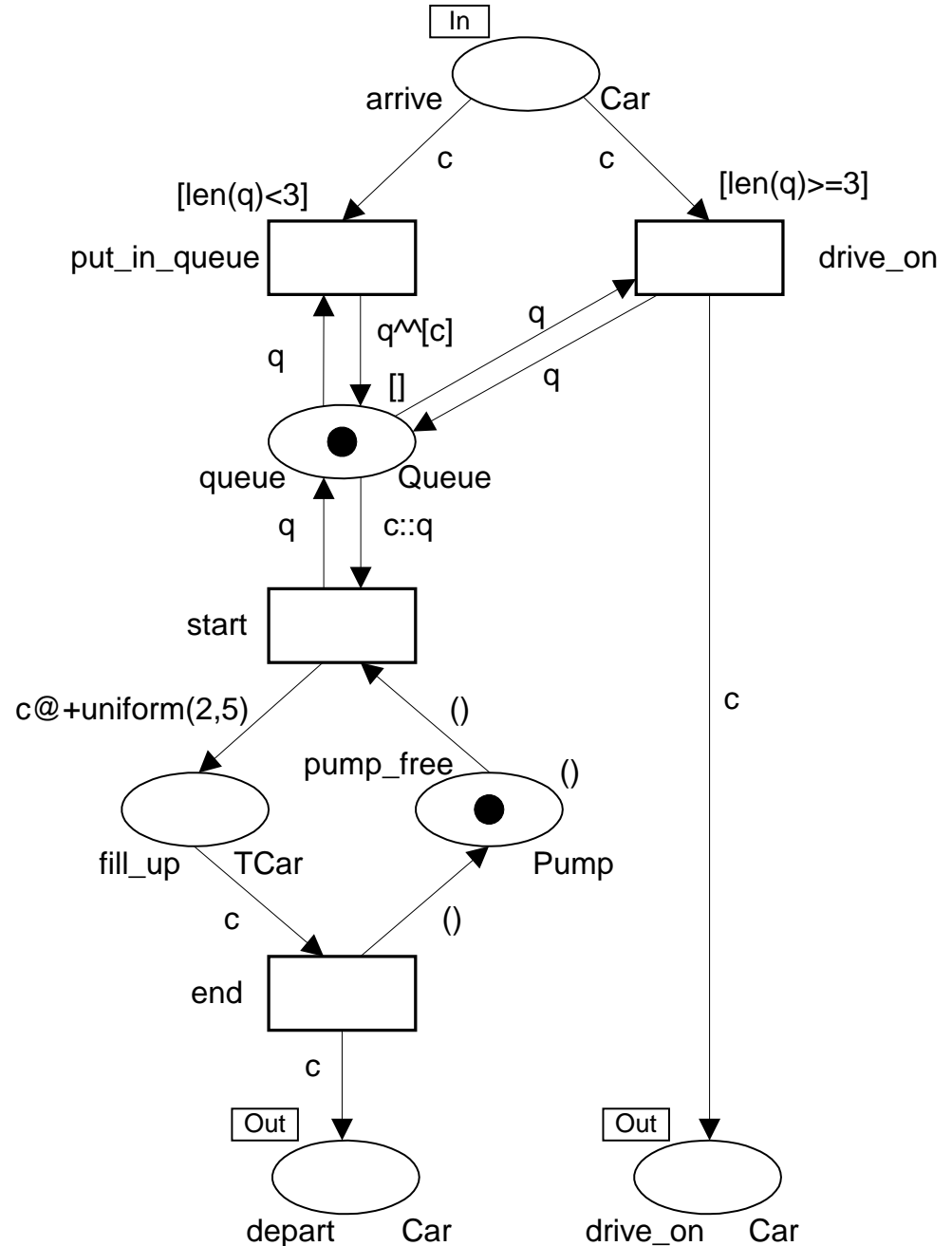
color Car = string



Subpage *gas_station*

```

color Car = string;
color Pump = unit;
color TCar = Car timed;
color Queue = list Car;
var c:Car;
var q:Queue;
fun len(q:Queue) = if q=[] then 0
  else 1+len(tl(q));
  
```



Interesting performance indicators:

- **Calculation of flow time (average, variance, maximum, minimum, service level, etc.).**
- **Calculation of waiting times (average, variance, maximum, minimum, service level, etc.).**
- **Calculation of lost sales (average).**
- **Probability of no space left.**
- **Probability of no cars waiting.**

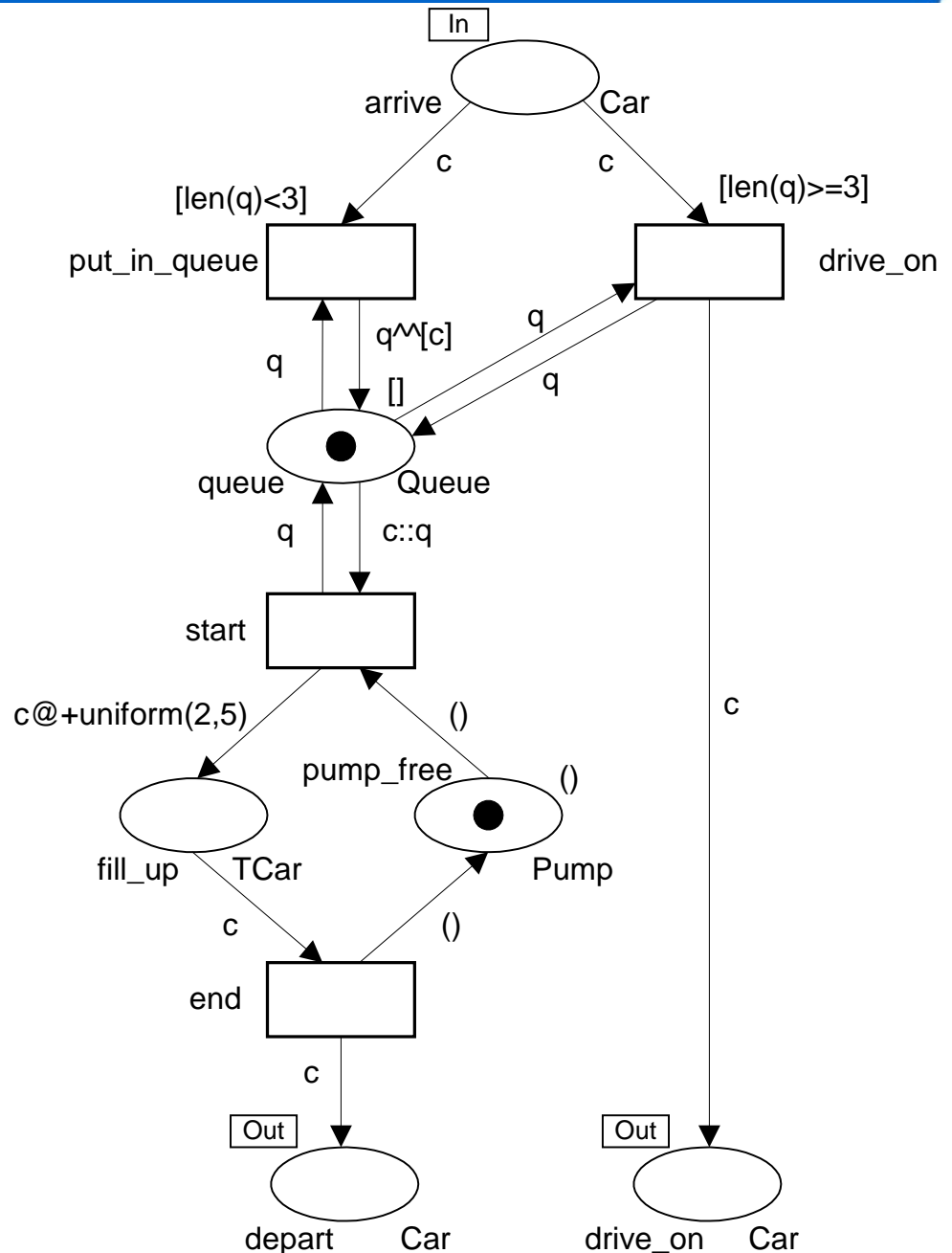
Alternatives

```

color Car = string;
color Pump = unit;
color TCar = Car timed;
color Queue = list Car;
var c:Car;
var q:Queue;
fun len(q:Queue) = if q=[] then 0
  else 1+len(tl(q));
  
```

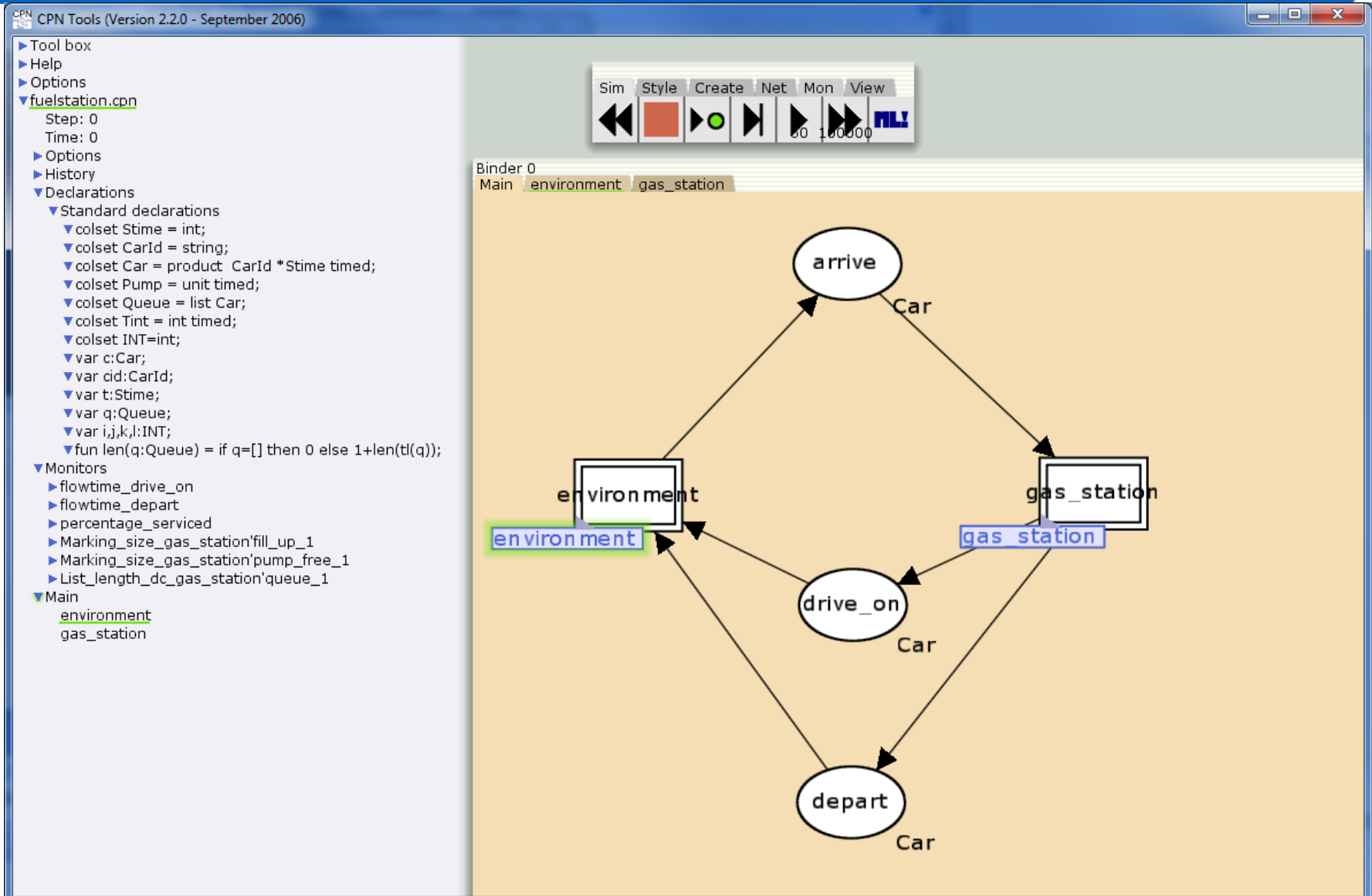
Model the following alternatives:

- 6 waiting spaces
- 2 pumps
- 1 faster pump



Experiments

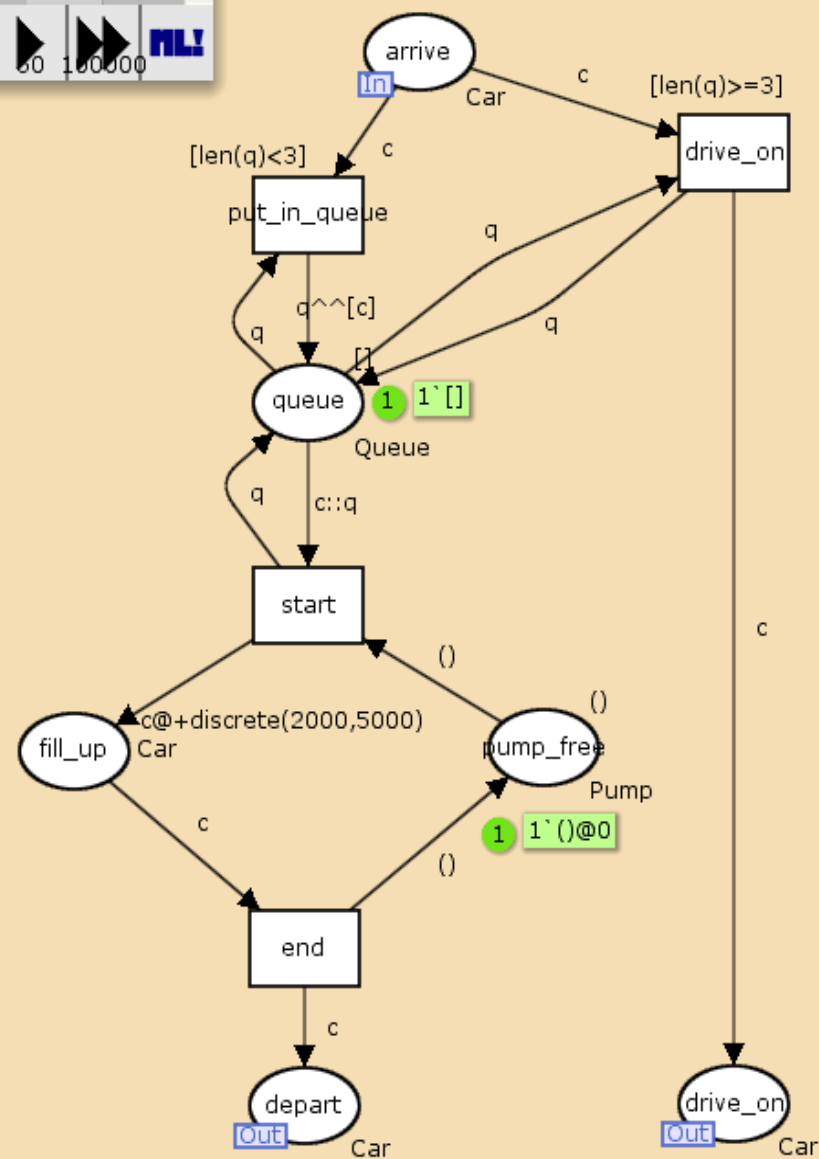
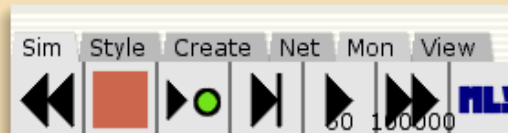
(note time dimension * 1000; not needed in CPN Tools Version 3)



▶ Tool box
 ▶ Help
 ▶ Options
 ▼ fuelstation.cpn
 Step: 0
 Time: 0
 ▶ Options
 ▶ History
 ▼ Declarations
 ▼ Standard declarations
 ▼ colset Stime = int;
 ▼ colset CarId = string;
 ▼ colset Car = product CarId * Stime timed;
 ▼ colset Pump = unit timed;
 ▼ colset Queue = list Car;
 ▼ colset Tint = int timed;
 ▼ colset INT=int;
 ▼ var c:Car;
 ▼ var cid:CarId;
 ▼ var t:Stime;
 ▼ var q:Queue;
 ▼ var i,j,k,l:INT;
 ▼ fun len(q:Queue) = if q=[] then 0 else 1+len(tl(q));
 ▼ Monitors
 ▶ flowtime_drive_on
 ▶ flowtime_depart
 ▶ percentage_served
 ▶ Marking_size_gas_station'fill_up_1
 ▶ Marking_size_gas_station'pump_free_1
 ▶ List_length_dc_gas_station'queue_1
 ▼ Main
 environment
 gas_station

Binder 0

Main environment gas_station



▶ Tool box
 ▶ Help
 ▶ Options
 ▼ fuelstation.cpn
 Step: 0
 Time: 0
 ▶ Options
 ▶ History
 ▼ Declarations
 ▼ Standard declarations
 ▼ colset Stime = int;
 ▼ colset CarId = string;
 ▼ colset Car = product CarId * Stime;
 ▼ colset Pump = unit timed;
 ▼ colset Queue = list Car;
 ▼ colset Tint = int timed;
 ▼ colset INT=int;
 ▼ var c:Car;
 ▼ var cid:CarId;
 ▼ var t:Stime;
 ▼ var q:Queue;
 ▼ var i,j,k,l:INT;
 ▼ fun len(q:Queue) = if q=[] then 0

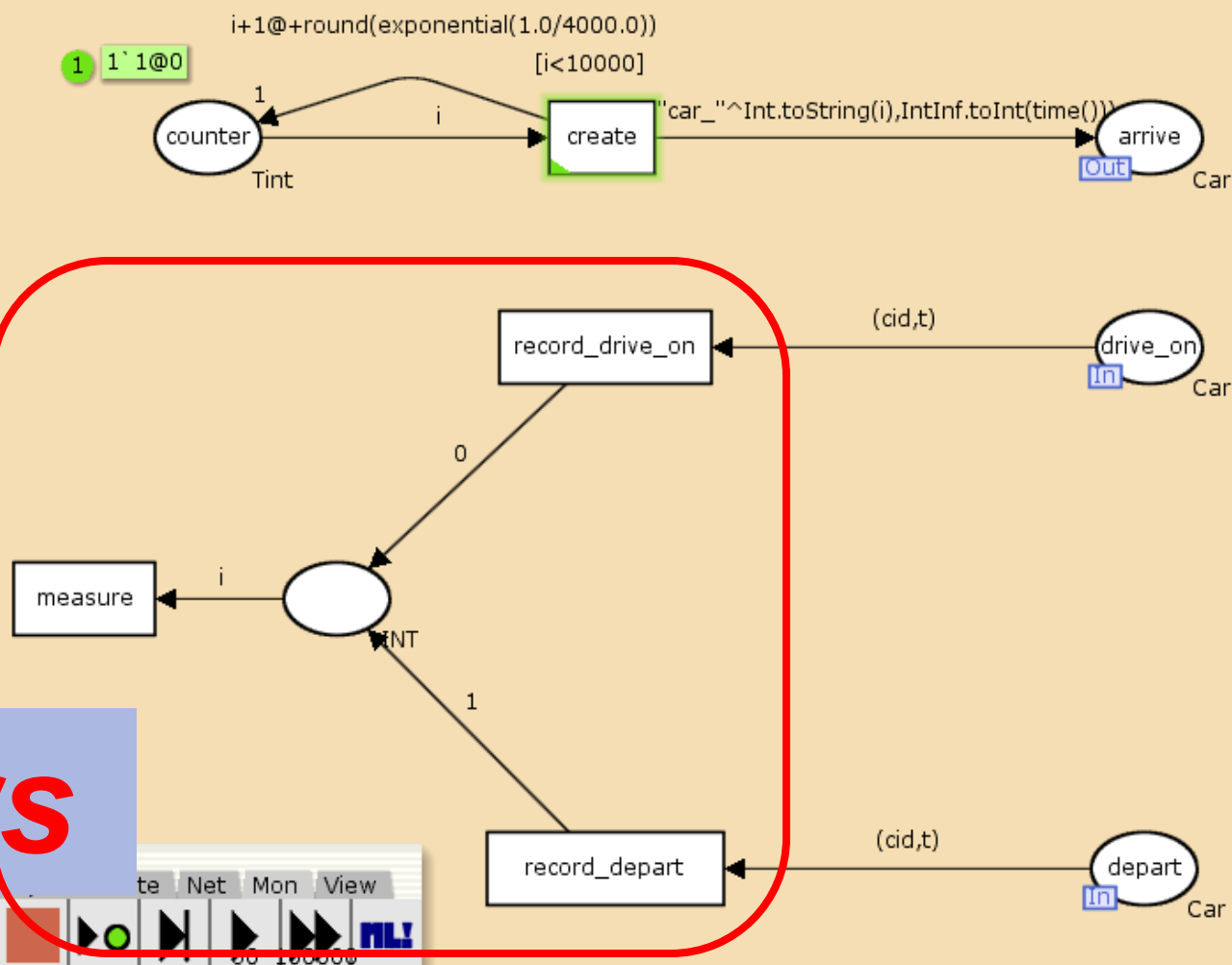
Monitors
 ▶ flowtime_drive_on
 ▶ flowtime_depart
 ▶ percentage_serviced
 ▶ Marking_size_gas_station'fill_up_1
 ▶ Marking_size_gas_station'pump_fr
 ▶ List_length_dc_gas_station'queue;

Main

Binder 0

Main environment gas_station

CPN'Replications.nreplications 10



monitors



- Tool box
- Help
- Options
- fuelstation.cpn

Step: 0
Time: 0

- Options
- History

Declarations

Standard declarations

- colset Stime = int;
- colset CarId = string;
- colset Car = product CarId * Stime timed;
- colset Pump = unit timed;
- colset Queue = list Car;
- colset Tint = int timed;
- colset INT = int;
- var c:Car;
- var cid:CarId;
- var t:Stime;
- var q:Queue;
- var i,j,k,l:INT;
- fun len(q:Queue) = if q=[] then 0 else 1+len(t

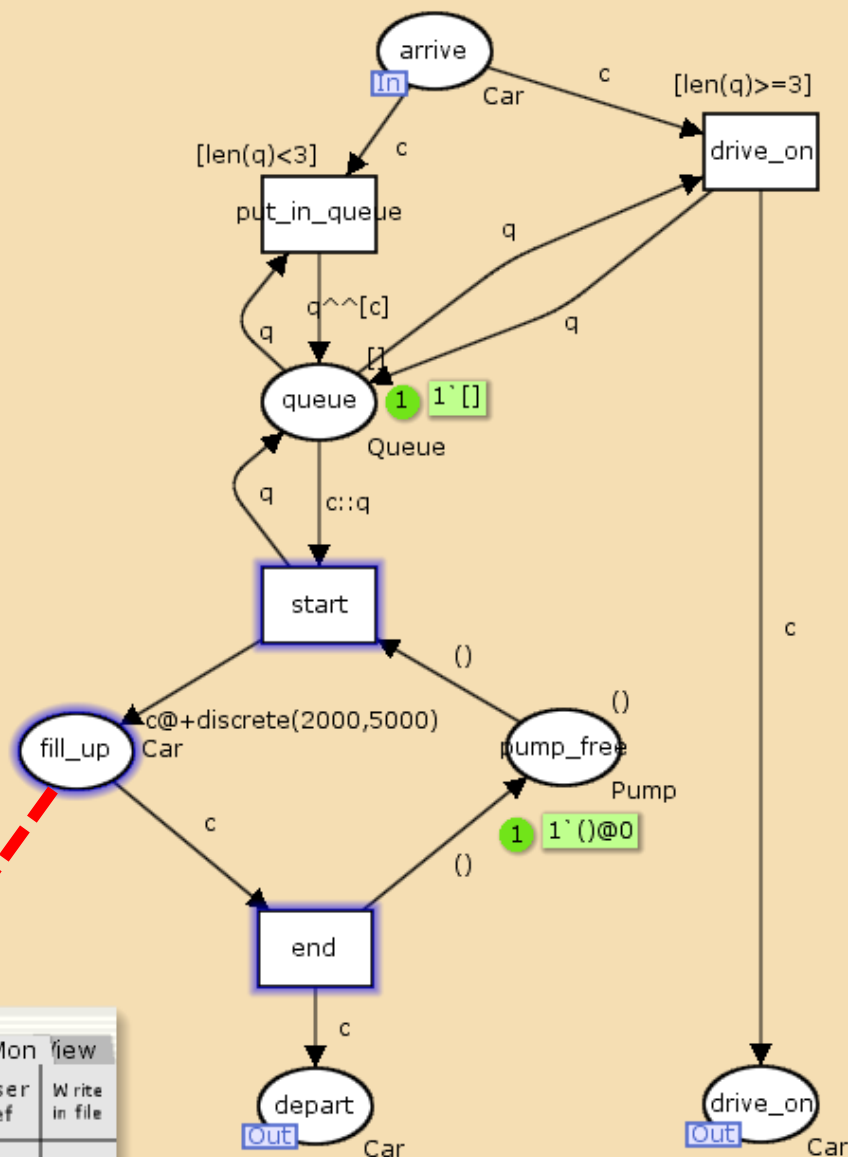
Monitors

- flowtime_drive_on
- flowtime_depart
- percentage_served
- Marking_size_gas_station'fill_up_1
 - Type: Marking size
 - Logging
 - Nodes ordered by pages
 - gas_station
 - end (transition)
 - fill_up (place)
 - start (transition)
- Marking_size_gas_station'pump_free_1
- List_length_dc_gas_station'queue_1

Main

- environment
- gas_station

Number of cars being served



Sim	Style	Create	Def	Mon	View
Data	Mark	Break	User	Write	
Coll	Size	point	def	in file	
LL	Coun	Place	Tran	Enab	
DC	Tran	Cont			

Number of pumps free

- Tool box
- Help
- Options
- fuelstation.cpn

Step: 0
Time: 0

- Options
- History
- Declarations

Standard declarations

```

colset Stime = int;
colset CarId = string;
colset Car = product CarId * Stime timed;
colset Pump = unit timed;
colset Queue = list Car;
colset Tint = int timed;
colset INT = int;
var c:Car;
var cid:CarId;
var t:Stime;
var q:Queue;
var i,j,k,l:INT;
fun len(q:Queue) = if q=[] then 0 else 1+len(t

```

Monitors

- flowtime_drive_on
- flowtime_depart
- percentage_served
- Marking_size_gas_station'fill_up_1
- Marking_size_gas_station'pump_free_1

Type: Marking size

☐ Logging

Nodes ordered by pages

gas_station

end (transition)

pump_free (place)

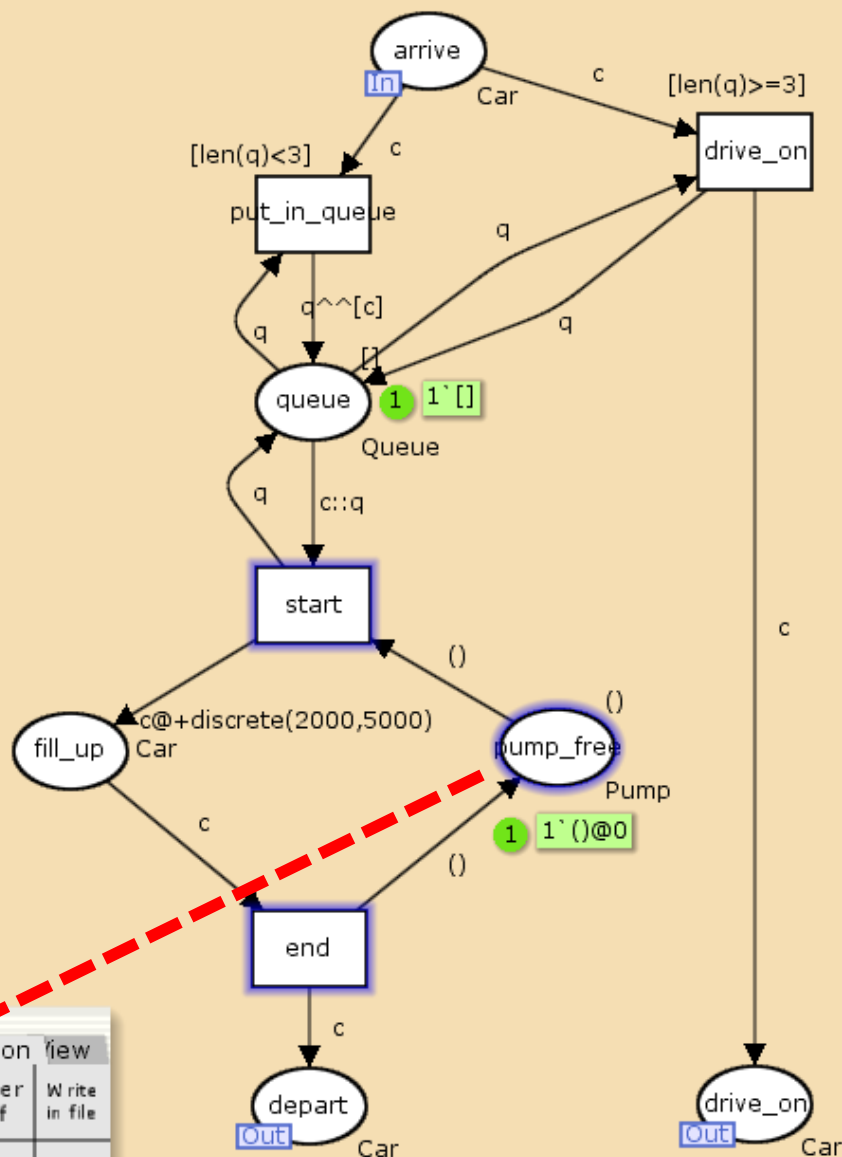
start (transition)

- List_length_dc_gas_station'queue_1

Main

environment

gas_station



Sim	ity	Create	del	Mon	view
Data	Mark	Break	User	Write	
Coll	Size	point	def	in file	
LL	Coun	Place	Tran		
DC	Tran	Cont	Enab		

Length of queue

- Tool box
- Help
- Options
- fuelstation.cpn

Step: 0
Time: 0

- Options
- History
- Declarations
 - Standard declarations
 - colset Stime = int;
 - colset CarId = string;
 - colset Car = product CarId * Stime timed;
 - colset Pump = unit timed;
 - colset Queue = list Car;
 - colset Tint = int timed;
 - colset INT = int;
 - var c:Car;
 - var cid:CarId;
 - var t:Stime;
 - var q:Queue;
 - var i,j,k,l:INT;
 - fun len(q:Queue) = if q=[] then 0 else 1+len(t

- Monitors
 - flowtime_drive_on
 - flowtime_depart
 - percentage_served
 - Marking_size_gas_station'fill_up_1
 - Marking_size_gas_station'pump_free_1
 - List_length_dc_gas_station'queue_1

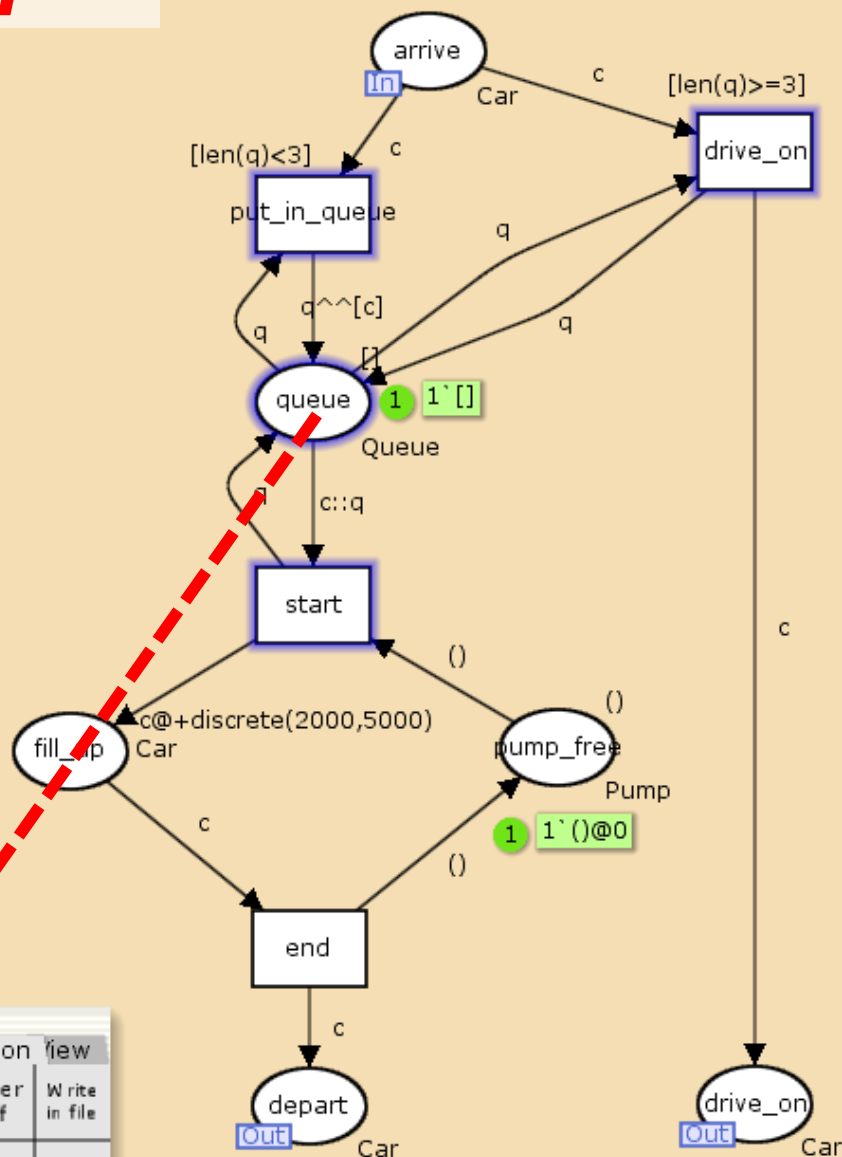
Type: List length data collection

- Logging
- Nodes ordered by pages
 - gas_station
 - drive_on (transition)
 - put_in_queue (transition)
 - queue (place)
 - start (transition)

- Main
 - environment
 - gas_station

Sim	Style	Create	Def	Mon	View
Data	Mark	Break	User		Write
Coll	Size	point	def		in file
LL	Coun	Place	Tran		
DC	Tran	Cont	Enab		

Binder 0
main_environment gas_station



```

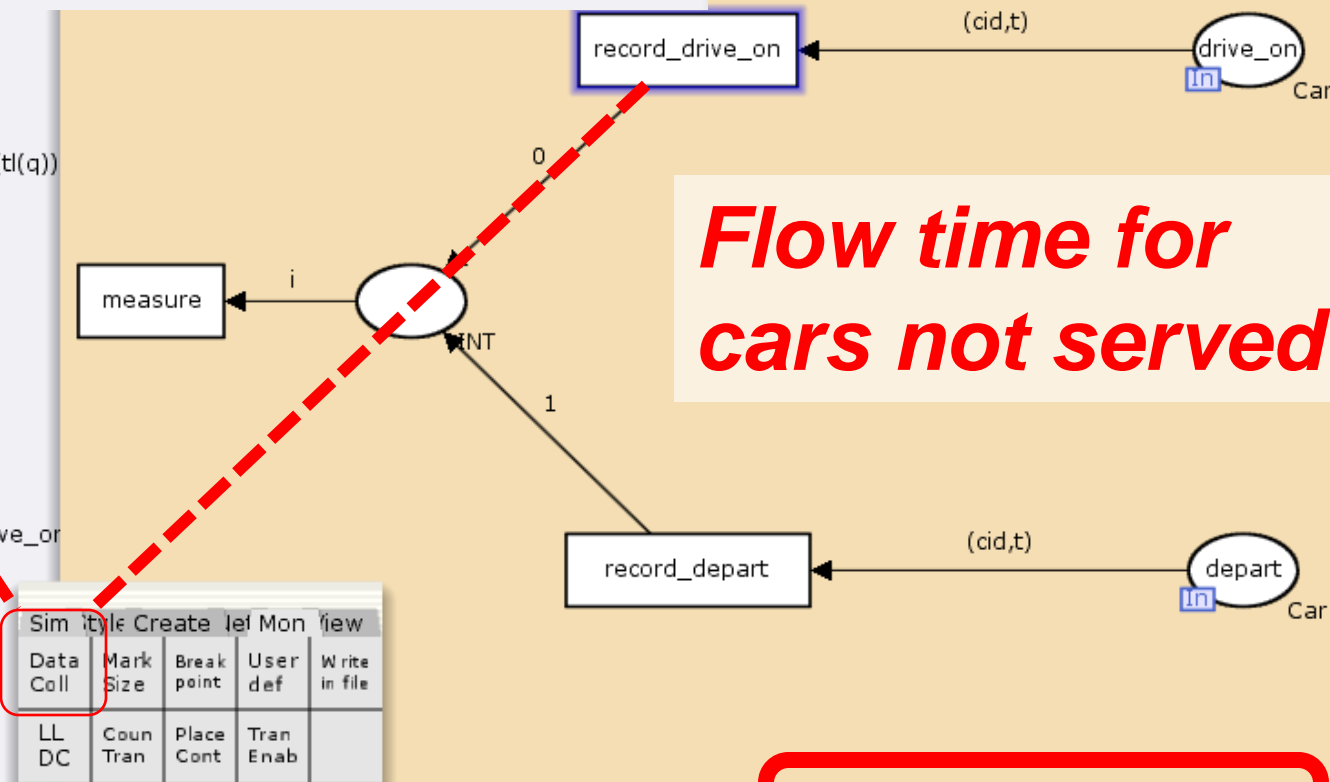
flowtime_drive_on
Type: Data collection
  Timed
  Logging
Nodes ordered by pages
  environment
    record_drive_on (transition)
  Predicate
  Observer
    fun obs (bindelem) =
    let
      fun obsBindElem (environment'record_drive_on (1, {cid,t})) = (IntInf.toInt(time())-t)
      | obsBindElem _ = ~1
    in
      obsBindElem bindelem
    end
  Init function
  Stop
var c:Car;
var cid:CarId;
var t:Time;
var q:Queue;
var i,j,k,l:INT;
fun len(q:Queue) = if q=[] then 0 else 1+len(tl(q))
Monitors
flowtime_drive_on
Type: Data collection
  Timed
  Logging
Nodes ordered by pages
  environment
    record_drive_on (transition)
  Predicate
  Observer
    fun obs (bindelem) =
    let
      fun obsBindElem (environment'record_drive_on (1, {cid,t})) = (IntInf.toInt(time())-t)
      | obsBindElem _ = ~1
    in
      obsBindElem bindelem
    end
  Init function
  Stop
percentage_served
Marking size gas station'fill up 1

```

```

fun obsBindElem (environment'record_drive_on (1, {cid,t})) = (IntInf.toInt(time())-t)
| obsBindElem _ = ~1

```



ol box
lp
tions
elstation.cpn

Step: 0

Time: 0

Options

History

Declarations

▼ Standard declarations

```

colset Stime = int;
colset CarId = string;
colset Car = product CarId * Stime timed;
colset Pump = unit timed;
colset Queue = list Car;
colset Tint = int timed;
colset INT=int;
var c:Car;
var cid:CarId;
var t:Stime;
var q:Queue;
var i,j,k,l:INT;
fun len(q:Queue) = if q=[] then 0 else 1+len(tl(q));

```

Monitors

► flowtime_drive_on

► flowtime_depart

▼ Type: Data collection

☐ Timed
☐ Logging

▼ Nodes ordered by pages

▼ environment
record_depart (transition)

► Predicate

▼ Observer

```

fun obs (bindelem) =
let
  fun obsBindElem (environment'record_depart (1, {cid,t})) = (IntInf.toInt(time())-t)
  | obsBindElem _ = ~1
in
  obsBindElem bindelem
end

```

► Init function

► Stop

► percentage_served

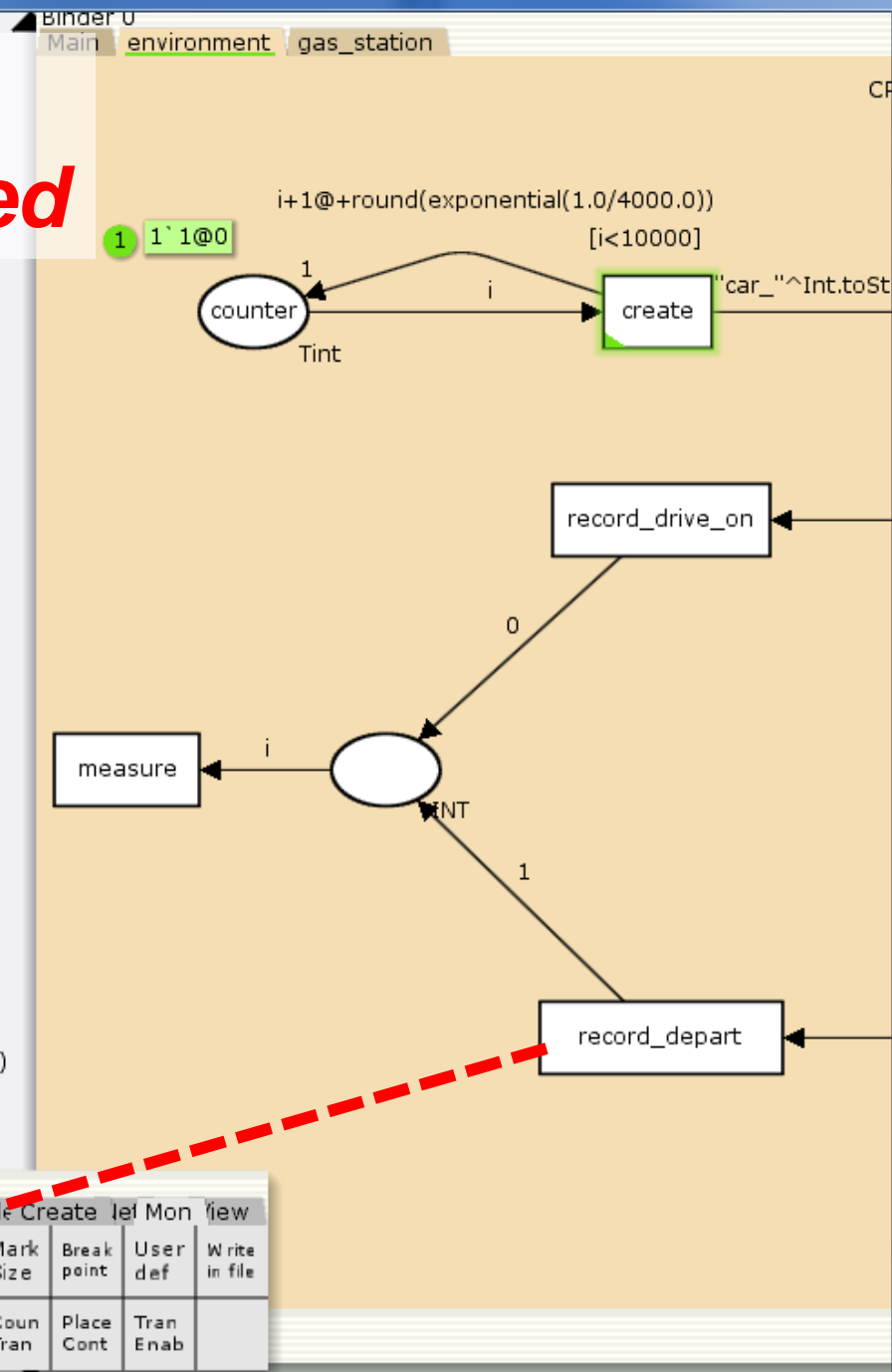
► Marking_size_gas_station'fill_up_1

► Marking_size_gas_station'pump_free_1

► List_length_dc_gas_station'queue_1

Main

Flow time for cars that have been served



Percentage of cars served

Control box
Help
Options
Simulation
Step: 0
Time: 0
Options
History
Declarations

▼ Standard declarations
 ▼ colset Stime = int;
 ▼ colset CarId = string;
 ▼ colset Car = product CarId * Stime timed;
 ▼ colset Pump = unit timed;
 ▼ colset Queue = list Car;
 ▼ colset Tint = int timed;
 ▼ colset INT=int;
 ▼ var c:Car;
 ▼ var cid:CarId;
 ▼ var t:Stime;
 ▼ var q:Queue;
 ▼ var i,j,k,l:INT;
 ▼ fun len(q:Queue) = if q=[] then 0 else 1+len(tl(q));

Monitors

► flowtime_drive_on
 ► flowtime_depart
 ▼ percentage_served
 ▼ Type: Data collection
 ☐ Timed
 ☐ Logging
 ▼ Nodes ordered by pages
 ▼ environment
 measure (transition)

► Predicate
 ▼ Observer
 fun obs (bindelem) =
 let
 fun obsBindElem (environment'measure (1, {i})) = i
 | obsBindElem _ = ~1
 in
 obsBindElem bindelem
 end

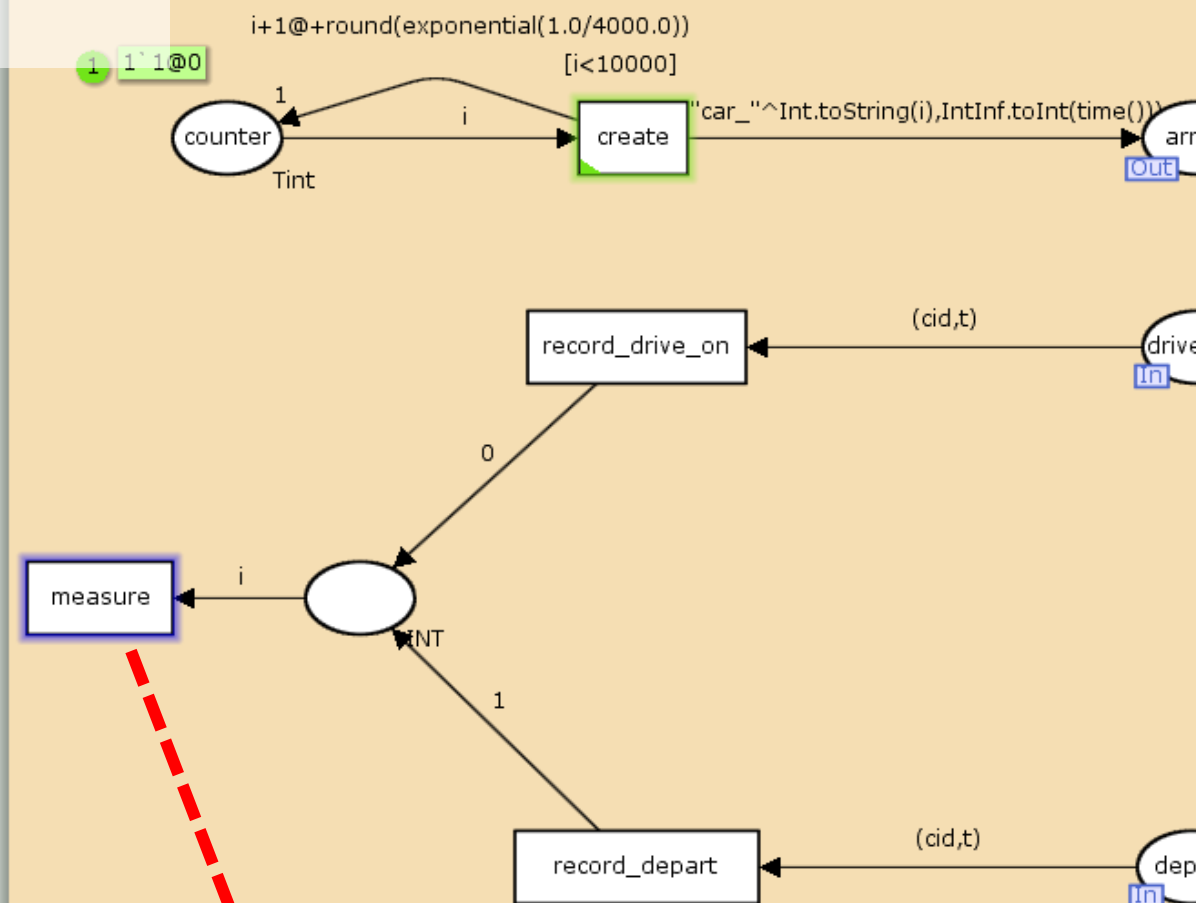
► Init function
 ► Stop
 ► Marking_size_gas_station'fill_up_1
 ► Marking_size_gas_station'pump_free_1
 ► List_length_dc_gas_station'queue_1

Main

Binder 0

Main environment gas_station

CPN'Replications.nreplications



Sim	Style	Create	let	Mon	view
Data	Mark	Break	User	Write	
Coll	Size	point	def	in file	
LL	Coun	Place	Tran		
DC	Tran	Cont	Enab		

Executes the specified number of transitions without showing the intermediate markings.



CPN Tools Simulation Performance Report
file:///D:/courses/BIS-2010/CPN/newsimulation/output/PerfReport.html

CPN Tools Simulation Performance Report
Net: D:\courses\BIS-2010\CPN\nsimulation\fuelstation.cpn

Note that these statistics have been calculated for data that is not necessarily independent or identically distributed.

Timed statistics				
Name	Count	Avrg	Min	Max
List_length_dc_gas_station'queue_1	19130	0.888529	0	3
Marking_size_gas_station'fill_up_1	18258	0.799214	0	1
Marking_size_gas_station'pump_free_1	18258	0.200786	0	1

Untimed statistics					
Name	Count	Sum	Avrg	Min	Max
flowtime_depart	9128	67178542	7359.612401	2005	17997
flowtime_drive_on	872	0	0.000000	0	0
percentage_serviced	10000	9128	0.912800	0	1

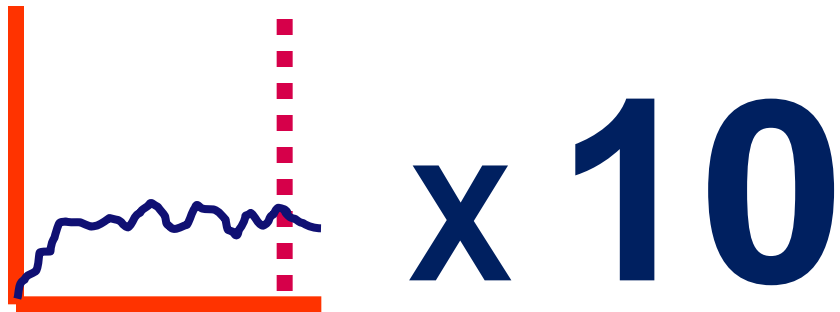
Simulation steps executed: 58256
Model time: 39803766

Generated: Sun Mar 28 19:04:26 2010

**Average
flow is
time 7.359**

Just one run ...

Subruns in CPN Tools



CPN'Replications.nreplications 10

CPN Tools Performance Report

Net D:\courses\BIS-2010\CPN\nwsimulation\fuelstation.cpn

Number of replications: 10

Statistics							
Name	Avrg	90% Half Length	95% Half Length	99% Half Length	StD	Min	Max
List_length_dc_gas_station'queue_1							
count_iid	19128.800000	25.132533	31.014615	44.561228	43.358454	19056	19181
max_iid	3.000000	0.000000	0.000000	0.000000	0.000000	3	3
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.891917	0.014062	0.017353	0.024932	0.024259	0.841207	0.924649
Marking_size_gas_station'fill_up_1							
count_iid	18255.600000	50.265065	62.029230	89.122456	86.716909	18110	18360
max_iid	1.000000	0.000000	0.000000	0.000000	0.000000	1	1
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.797933	0.003853	0.004755	0.006832	0.006648	0.790479	0.810488
Marking_size_gas_station'pump_free_1							
count_iid	18255.600000	50.265065	62.029230	89.122456	86.716909	18110	18360
max_iid	1.000000	0.000000	0.000000	0.000000	0.000000	1	1
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.202067	0.003853	0.004755	0.006832	0.006648	0.189512	0.209521

Average queue length [0.878,0.906]

Average # pumps busy [0.794,0.801]

Average # pumps free [0.198,0.206]

avrg_iid	0.202067	0.003853	0.004755	0.006832	0.006648	0.189512	0.209521
flowtime_depart							
count_iid	9126.800000	25.132533	31.014615	44.561228	43.358454	9054	9179
max_iid	18192.300000	181.666042	224.183626	322.102911	313.408874	17593	18615
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	67592080.300000	438346.885657	540938.709960	777210.790172	756232.713539	66432681	68943753
avrg_iid	7406.069819	53.540469	66.071218	94.929910	92.367611	7237.463885	7523.325295
flowtime_drive_on							
count_iid	873.200000	25.132533	31.014615	44.561228	43.358454	821	946
max_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
percentage_served							
count_iid	10000.000000	0.000000	0.000000	0.000000	0.000000	10000	10000
max_iid	10000.000000	0.000000	0.000000	0.000000	0.000000	10000	10000
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	9126.800000	25.132533	31.014615	44.561228	43.358454	9054	9179
avrg_iid	0.912680	0.002513	0.003101	0.004456	0.004336	0.905400	0.917900

Average flow time [7.352,7.460]

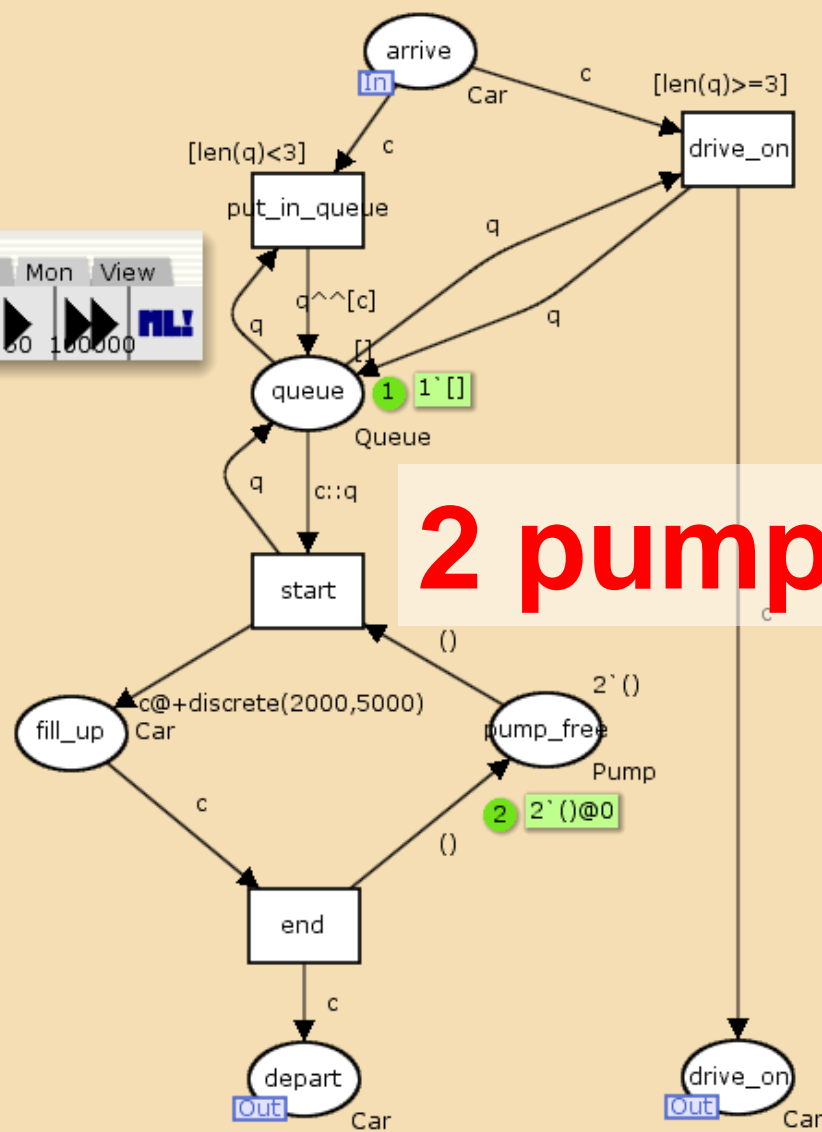
Average fraction served [0.910,0.915]

Results

	Average flow time	Average fraction served
Base case	[7.352,7.460]	[0.910,0.915]

- ▶ Tool box
- ▶ Help
- ▶ Options
- ▼ fuelstation2pumps.cpn
 - Step: 0
 - Time: 0
 - ▶ Options
 - ▶ History
 - ▼ Declarations
 - ▼ Standard declarations
 - ▼ colset Stime = int;
 - ▼ colset CarId = string;
 - ▼ colset Car = product CarId * Stime timed;
 - ▼ colset Pump = unit timed;
 - ▼ colset Queue = list Car;
 - ▼ colset Tint = int timed;
 - ▼ colset INT=int;
 - ▼ var c:Car;
 - ▼ var cid:CarId;
 - ▼ var t:Stime;
 - ▼ var q:Queue;
 - ▼ var i,j,k,l:INT;
 - ▼ fun len(q:Queue) = if q=[] then 0 else 1+
 - ▼ Monitors
 - ▶ flowtime_drive_on
 - ▶ flowtime_depart
 - ▶ percentage_served
 - ▶ Marking_size_gas_station'fill_up_1
 - ▶ Marking_size_gas_station'pump_free_1
 - ▼ List_length_dc_gas_station'queue_1
 - ▼ Type: List length data collection
 - ☐ Logging
 - ▶ Nodes ordered by pages
 - ▼ Main
 - environment
 - gas_station

Binder 0
Main environment gas_station



CPN Tools Performance Report
 Net D:\courses\BIS-2010\CPN\newsimulation\fuelstation2pumps.cpn
 Number of replications: 10

Statistics							
Name	Avrg	90% Half Length	95% Half Length	99% Half Length	Std	Min	Max
List_length_dc_gas_station'queue_1							
count_iid	19963.900000	3.318017	4.094574	5.883009	5.724218	19955	19972
max_iid	3.000000	0.000000	0.000000	0.000000	0.000000	3	3
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.107718	0.002875	0.003548	0.005098	0.004960	0.100403	0.116024
Marking_size_gas_station'fill_up_1							
count_iid	19925.800000	6.636034	8.189148	11.766017	11.448435	19908	19942
max_iid	2.000000	0.000000	0.000000	0.000000	0.000000	2	2
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.872762	0.005708	0.007044	0.010121	0.009848	0.853645	0.883708
Marking_size_gas_station'pump_free_1							
count_iid	19925.800000	6.636034	8.189148	11.766017	11.448435	19908	19942
max_iid	2.000000	0.000000	0.000000	0.000000	0.000000	2	2
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	1.127238	0.005708	0.007044	0.010121	0.009848	1.116292	1.146355

avrg_iid	1.127238	0.005708	0.007044	0.010121	0.009848	1.116292	1.146355
flowtime_depart							
count_iid	9961.900000	3.318017	4.094574	5.883009	5.724218	9953	9970
max_iid	11600.600000	343.702375	424.143357	609.401375	592.952724	10785	12662
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	39154411.600000	131855.985224	162715.896659	233787.207843	227476.942953	38778392	39503646
avrg_iid	3930.421027	13.717449	16.927915	24.321718	23.665238	3893.412851	3966.627774
flowtime_drive_on							
count_iid	38.100000	3.318017	4.094574	5.883009	5.724218	30	47
max_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
avrg_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
percentage_served							
count_iid	10000.000000	0.000000	0.000000	0.000000	0.000000	10000	10000
max_iid	10000.000000	0.000000	0.000000	0.000000	0.000000	10000	10000
min_iid	0.000000	0.000000	0.000000	0.000000	0.000000	0	0
sum_iid	9961.900000	3.318017	4.094574	5.883009	5.724218	9953	9970
avrg_iid	0.996190	0.000332	0.000409	0.000588	0.000572	0.995300	0.997000

Average flow time [3.916,3.944]

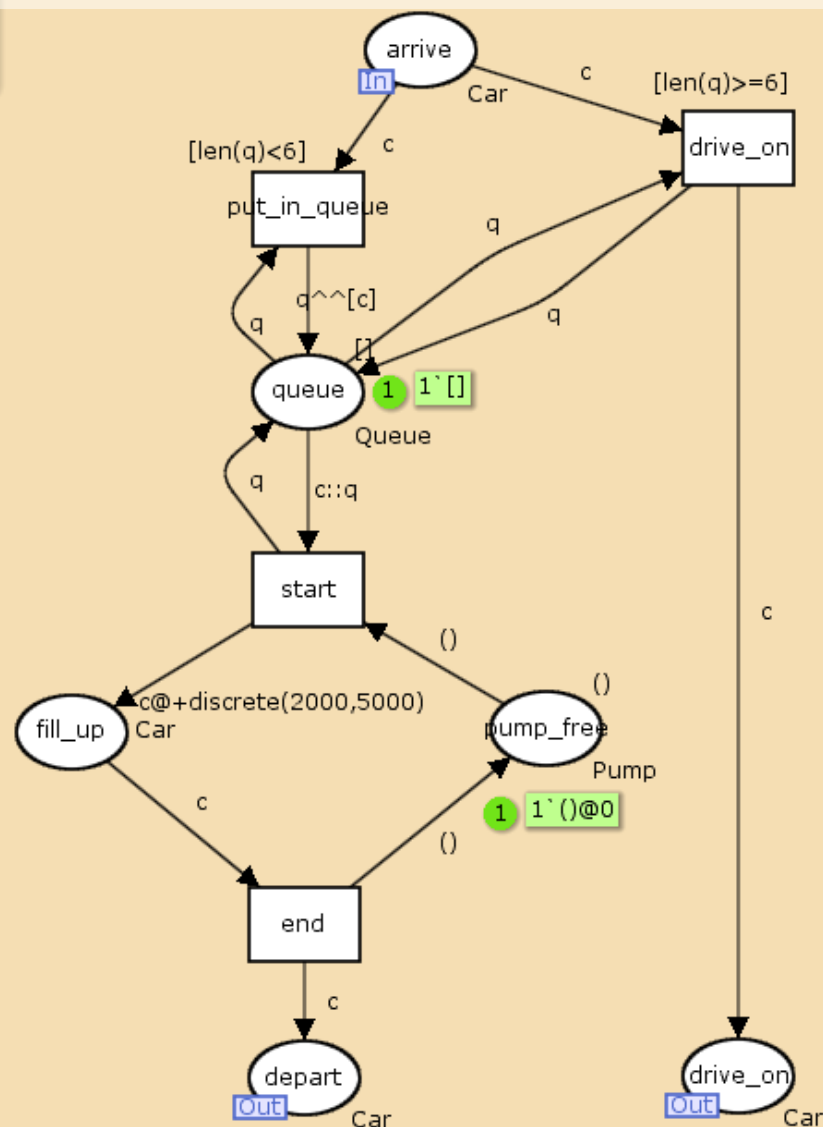
Average fraction served [0.996,0.997]

Results

	Average flow time	Average fraction served
Base case	[7.352,7.460]	[0.910,0.915]
Two pumps	[3.916,3.944]	[0.996,0.997]

- ```
Binder 0
Main environment
```

Sim Style Create Net Mon View



## CPN Tools Performance Report

Net D:\courses\BIS-2010\CPN\nwsimulation\fuelstation5places.cpn

Number of replications: 10

## Statistics

| Name                               | Avrg         | 90% Half Length | 95% Half Length | 99% Half Length | StD       | Min      | Max      |
|------------------------------------|--------------|-----------------|-----------------|-----------------|-----------|----------|----------|
| List_length_dc_gas_station'queue_1 |              |                 |                 |                 |           |          |          |
| count_iid                          | 19680.900000 | 18.247226       | 22.517854       | 32.353238       | 31.479976 | 19632    | 19746    |
| max_iid                            | 6.000000     | 0.000000        | 0.000000        | 0.000000        | 0.000000  | 6        | 6        |
| min_iid                            | 0.000000     | 0.000000        | 0.000000        | 0.000000        | 0.000000  | 0        | 0        |
| avrg_iid                           | 1.730337     | 0.039569        | 0.048830        | 0.070159        | 0.068265  | 1.648615 | 1.853813 |

## Marking\_size\_gas\_station'fill\_up\_1

|           |              |           |           |           |           |          |          |
|-----------|--------------|-----------|-----------|-----------|-----------|----------|----------|
| count_iid | 19359.800000 | 36.494453 | 45.035707 | 64.706476 | 62.959952 | 19262    | 19490    |
| max_iid   | 1.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 1        | 1        |
| min_iid   | 0.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 0        | 0        |
| avrg_iid  | 0.845927     | 0.004984  | 0.006150  | 0.008837  | 0.008598  | 0.830000 | 0.860005 |

## Marking\_size\_gas\_station'pump\_free\_1

|           |              |           |           |           |           |          |          |
|-----------|--------------|-----------|-----------|-----------|-----------|----------|----------|
| count_iid | 19359.800000 | 36.494453 | 45.035707 | 64.706476 | 62.959952 | 19262    | 19490    |
| max_iid   | 1.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 1        | 1        |
| min_iid   | 0.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 0        | 0        |
| avrg_iid  | 0.154073     | 0.004984  | 0.006150  | 0.008837  | 0.008598  | 0.139995 | 0.170000 |

Average queue length [1.691,1.770]

Average # pumps busy [0.841,0.851]

Average # pumps free [0.149,0.159]



|                          |                  |                |                |                |                |              |              |
|--------------------------|------------------|----------------|----------------|----------------|----------------|--------------|--------------|
| avrg_iid                 | 0.154073         | 0.004984       | 0.006150       | 0.008837       | 0.008598       | 0.139995     | 0.170000     |
| <b>flowtime_depart</b>   |                  |                |                |                |                |              |              |
| count_iid                | 9678.900000      | 18.247226      | 22.517854      | 32.353238      | 31.479976      | 9630         | 9744         |
| max_iid                  | 30006.900000     | 299.730152     | 369.879763     | 531.436441     | 517.092180     | 29374        | 31188        |
| min_iid                  | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0            | 0            |
| sum_iid                  | 103039273.400000 | 1101932.891192 | 1359832.078492 | 1953781.721972 | 1901046.243765 | 100848139    | 106400048    |
| avrg_iid                 | 10646.221288     | 127.439617     | 157.265911     | 225.956768     | 219.857858     | 10395.074418 | 11048.810800 |
| <b>flowtime_drive_on</b> |                  |                |                |                |                |              |              |
| count_iid                | 321.100000       | 18.247226      | 22.517854      | 32.353238      | 31.479976      | 256          | 370          |
| max_iid                  | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0            | 0            |
| min_iid                  | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0            | 0            |
| sum_iid                  | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0            | 0            |
| avrg_iid                 | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0.000000     | 0.000000     |
| <b>percentage_served</b> |                  |                |                |                |                |              |              |
| count_iid                | 10000.000000     | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 10000        | 10000        |
| max_iid                  | 10000.000000     | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 10000        | 10000        |
| min_iid                  | 0.000000         | 0.000000       | 0.000000       | 0.000000       | 0.000000       | 0            | 0            |
| sum_iid                  | 9678.900000      | 18.247226      | 22.517854      | 32.353238      | 31.479976      | 9630         | 9744         |
| avrg_iid                 | 0.967890         | 0.001825       | 0.002252       | 0.003235       | 0.003148       | 0.963000     | 0.974400     |

Average flow time [10.518, 10.774]

Average fraction served [0.966, 0.970]

# Results

|            | Average flow time | Average fraction served |
|------------|-------------------|-------------------------|
| Base case  | [7.352,7.460]     | [0.910,0.915]           |
| Two pumps  | [3.916,3.944]     | [0.996,0.997]           |
| Six places | [10.518,10.774]   | [0.966,0.970]           |

Tool box  
Help  
Options  
fuelstationfastpump.cpn

Step: 0  
Time: 0

Sim Style Create Net Mon View

ML

Options  
History  
Declarations

Standard declarations

- colset Stime = int;
- colset CarId = string;
- colset Car = product CarId \* Stime timed;
- colset Pump = unit timed;
- colset Queue = list Car;
- colset Tint = int timed;
- colset INT=int;
- var c:Car;
- var cid:CarId;
- var t:Stime;
- var q:Queue;
- var i,j,k,l:INT;
- fun len(q:Queue) = if q=[] then 0 else 1+len(tl(q));

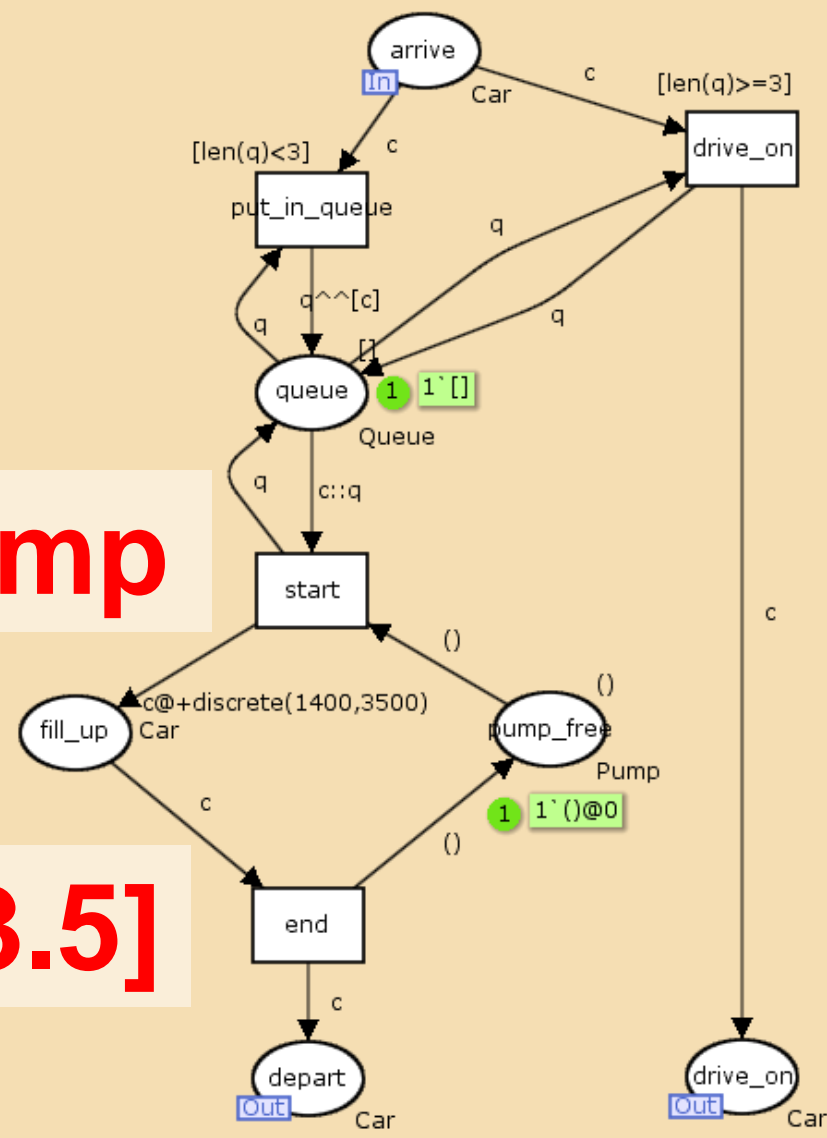
Monitors

- flowtime\_drive\_on
- flowtime\_depart
- percentage\_served
- Marking\_size\_gas\_station'fill\_up\_1
  - Type: Marking size
  - Nodes ordered by pages
- Marking\_size\_gas\_station'pump\_free\_1
  - Type: Marking size
  - Nodes ordered by pages
- List\_length\_dc\_gas\_station'queue\_1
  - Type: List length data collection
  - Nodes ordered by pages

Main  
environment  
gas\_station

faster pump

[2-5] => [1.4-3.5]



## CPN Tools Performance Report

Net D:\courses\BIS-2010\CPN\nwsimulation\fuelstationfastpump.cpn

Number of replications: 10

## Statistics

| Name                               | Avrg         | 90% Half Length | 95% Half Length | 99% Half Length | Std       | Min      | Max      |
|------------------------------------|--------------|-----------------|-----------------|-----------------|-----------|----------|----------|
| List_length_dc_gas_station'queue_1 |              |                 |                 |                 |           |          |          |
| count_iid                          | 19779.800000 | 15.691641       | 19.364153       | 27.822059       | 27.071100 | 19731    | 19827    |
| max_iid                            | 3.000000     | 0.000000        | 0.000000        | 0.000000        | 0.000000  | 3        | 3        |
| min_iid                            | 0.000000     | 0.000000        | 0.000000        | 0.000000        | 0.000000  | 0        | 0        |
| avrg_iid                           | 0.388141     | 0.010423        | 0.012863        | 0.018481        | 0.017982  | 0.358039 | 0.419027 |

## Marking\_size\_gas\_station'fill\_up\_1

|           |              |           |           |           |           |          |          |
|-----------|--------------|-----------|-----------|-----------|-----------|----------|----------|
| count_iid | 19557.600000 | 31.383282 | 38.728306 | 55.644117 | 54.142200 | 19460    | 19652    |
| max_iid   | 1.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 1        | 1        |
| min_iid   | 0.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 0        | 0        |
| avrg_iid  | 0.600104     | 0.004608  | 0.005687  | 0.008170  | 0.007950  | 0.586972 | 0.612357 |

## Marking\_size\_gas\_station'pump\_free\_1

|           |              |           |           |           |           |          |          |
|-----------|--------------|-----------|-----------|-----------|-----------|----------|----------|
| count_iid | 19557.600000 | 31.383282 | 38.728306 | 55.644117 | 54.142200 | 19460    | 19652    |
| max_iid   | 1.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 1        | 1        |
| min_iid   | 0.000000     | 0.000000  | 0.000000  | 0.000000  | 0.000000  | 0        | 0        |
| avrg_iid  | 0.399896     | 0.004608  | 0.005687  | 0.008170  | 0.007950  | 0.387643 | 0.413028 |

Average queue length [0.378,0.399]

Average # pumps busy [0.595,0.605]

Average # pumps free [0.395,0.405]

|                          |                 |               |               |               |               |             |             |
|--------------------------|-----------------|---------------|---------------|---------------|---------------|-------------|-------------|
| avrg_iid                 | 0.399896        | 0.004608      | 0.005687      | 0.008170      | 0.007950      | 0.387643    | 0.413028    |
| <b>flowtime_depart</b>   |                 |               |               |               |               |             |             |
| count_iid                | 9777.800000     | 15.691641     | 19.364153     | 27.822059     | 27.071100     | 9729        | 9825        |
| max_iid                  | 12359.400000    | 177.506636    | 219.050743    | 314.728079    | 306.233099    | 11721       | 12731       |
| min_iid                  | 1.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0           | 0           |
| sum_iid                  | 39430335.100000 | 272477.110796 | 336248.349493 | 483115.444674 | 470075.439377 | 38765863    | 40232718    |
| avrg_iid                 | 4032.742007     | 32.360712     | 39.934495     | 57.377149     | 55.828454     | 3945.634911 | 4135.339500 |
| <b>flowtime_drive_on</b> |                 |               |               |               |               |             |             |
| count_iid                | 222.200000      | 15.691641     | 19.364153     | 27.822059     | 27.071100     | 175         | 271         |
| max_iid                  | 0.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0           | 0           |
| min_iid                  | 0.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0           | 0           |
| sum_iid                  | 0.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0           | 0           |
| avrg_iid                 | 0.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0.000000    | 0.000000    |
| <b>percentage_served</b> |                 |               |               |               |               |             |             |
| count_iid                | 10000.000000    | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 10000       | 10000       |
| max_iid                  | 1.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 1           | 1           |
| min_iid                  | 0.000000        | 0.000000      | 0.000000      | 0.000000      | 0.000000      | 0           | 0           |
| sum_iid                  | 9777.800000     | 15.691641     | 19.364153     | 27.822059     | 27.071100     | 9729        | 9825        |
| avrg_iid                 | 0.977780        | 0.001569      | 0.001936      | 0.002782      | 0.002707      | 0.972900    | 0.982500    |

Average flow time [4.000,4.065]

Average fraction served [0.976,0.979]

# Results

|             | Average flow time      | Average fraction served |
|-------------|------------------------|-------------------------|
| Base case   | [7.352,7.460]          | <b>[0.910,0.915]</b>    |
| Two pumps   | <b>[3.916,3.944]</b>   | [0.996,0.997]           |
| Six places  | <b>[10.518,10.774]</b> | [0.966,0.970]           |
| Faster pump | <b>[4.000,4.065]</b>   | <b>[0.976,0.979]</b>    |

# Insights obtained from simulation

- Adding a pump significantly reduces the flow time (from approx. 7.4 to approx. 3.9 minutes) and reduces the percentage not served (from approx. 9% to approx. 1%).
- Adding more waiting places significantly increases the flow time (from approx. 7.4 to approx. 10.6 minutes) but reduces the percentage not served (approx. 9% to approx. 3%).
- Installing a faster pump significantly reduces the flow time (from approx. 7.4 to approx. 4.0 minutes) and reduces the percentage not served (from approx. 9% to approx. 3%).

# Analytical models versus Simulation models





# Example: M/M/1 queue

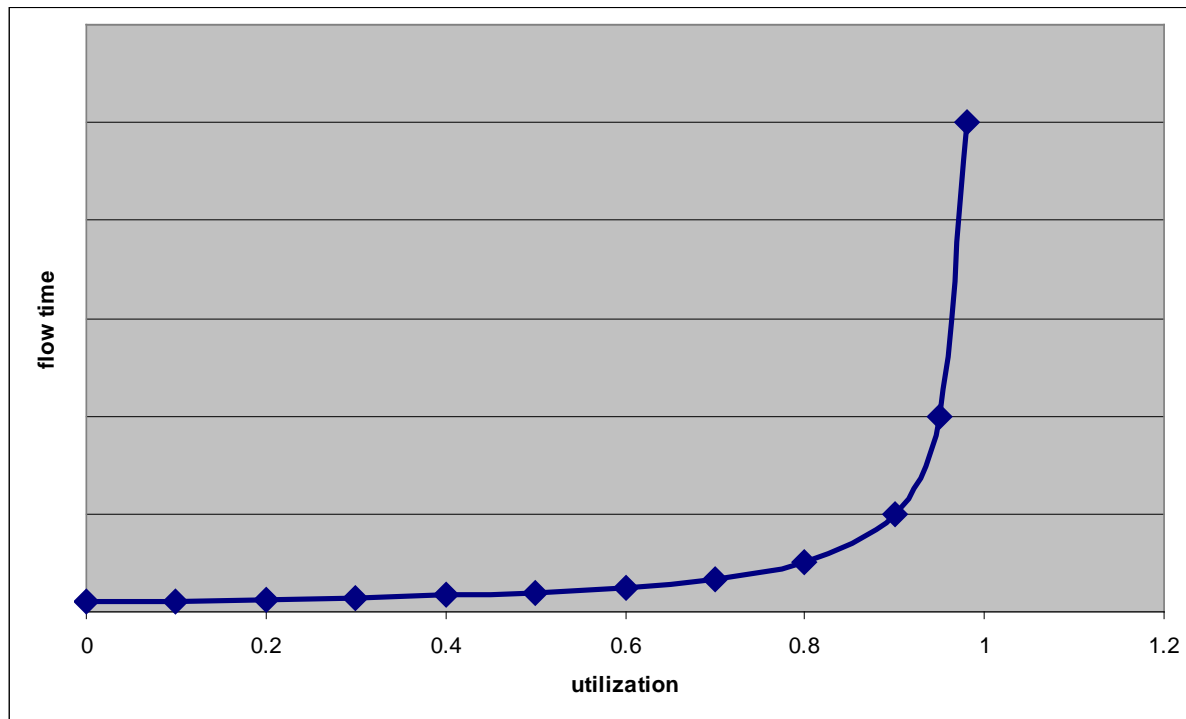
- arrival rate  $\lambda$  (average interarrival time =  $1/\lambda$ )
- service rate  $\mu$  (average interarrival time =  $1/\mu$ )
- utilization  $\rho = \lambda/\mu$
- average nof cases in system  $L = \rho/(1 - \rho)$
- average flow time  $S = 1/(\mu - \lambda)$
- Example:
  - $\lambda = 1/100$  and  $\mu = 1/50$
  - $\rho = 0.5$
  - $L = 1$
  - $S = 100$

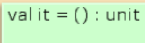
# M/M/1 queue

- $\lambda = 1/100$
- $\mu = 1/50$
- $\rho = 0.5$
- $L = 1$
- $S = 100$

- $\lambda = 1/100$
- $\mu = 1/80$
- $\rho = 0.8$
- $L = 4$
- $S = 400$

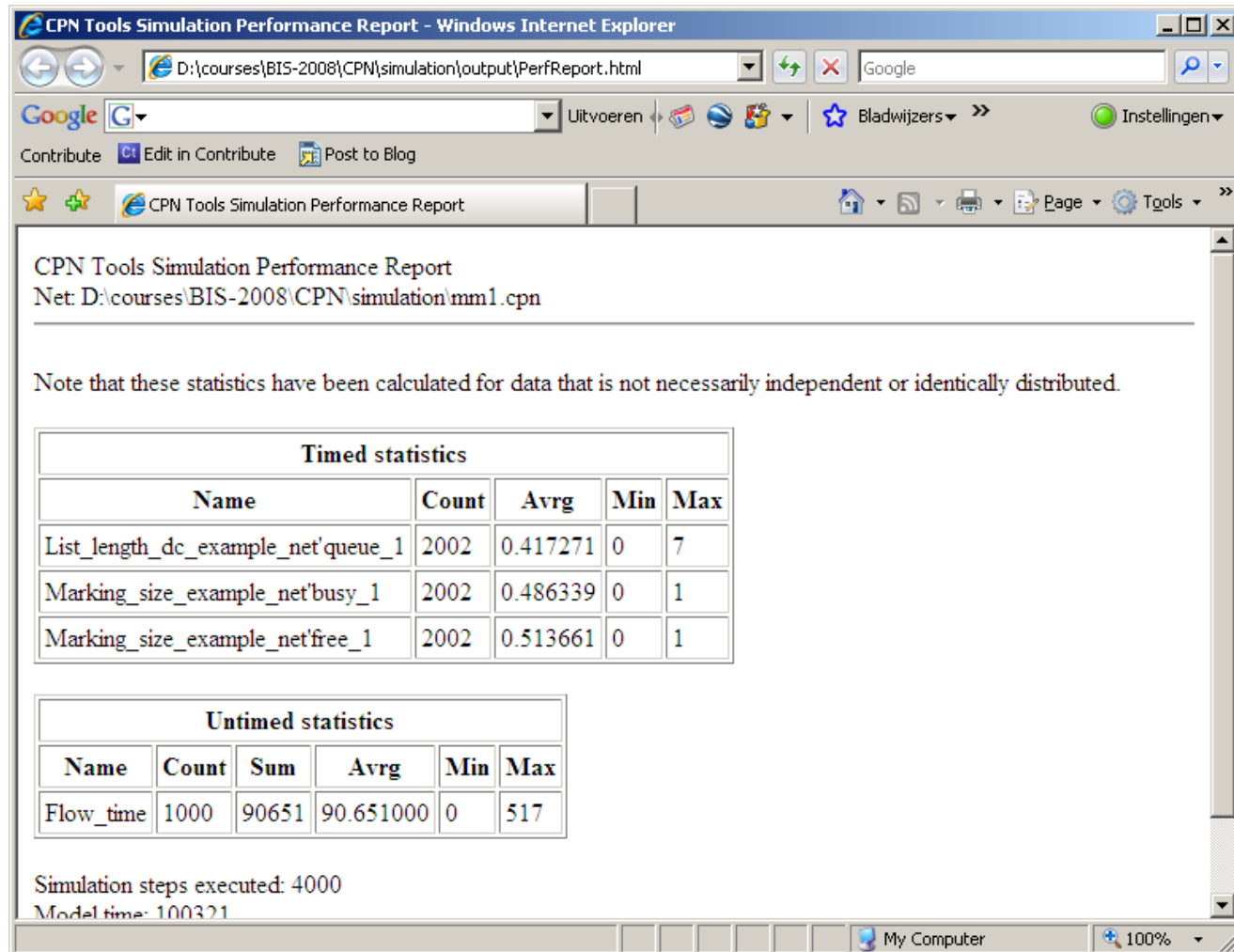
- $\lambda = 1/100$
- $\mu = 1/99$
- $\rho = 0.99$
- $L = 99$
- $S = 9900$







# Single run



90% [96.703-4.01059,96703+4.0159]

- $\lambda = 1/100$
- $\mu = 1/50$
- $\rho = 0.5$
- $L = 1$
- $S = 100$

CPN Tools Performance Report  
Net: D:\courses\BIS-2008\CPN\simulation\output\reps\_2\PerfReportIID.html  
Number of replications: 10

## Statistics

| Name      | Avrg         | 90% Half Length | 95% Half Length | 99% Half Length | Std         | Min       | Max        |
|-----------|--------------|-----------------|-----------------|-----------------|-------------|-----------|------------|
| Flow_time |              |                 |                 |                 |             |           |            |
| count_iid | 1000.000000  | 0.000000        | 0.000000        | 0.000000        | 0.000000    | 1000      | 1000       |
| max_iid   | 587.300000   | 68.02365        | 84.534834       | 121.458094      | 118.179760  | 440       | 780        |
| min_iid   | 0.000000     | 0.000000        | 0.000000        | 0.000000        | 0.000000    | 0         | 0          |
| sum_iid   | 96703.900000 | 4010.589995     | 4949.238717     | 7110.975169     | 6919.039359 | 86441     | 107560     |
| avrg_iid  | 96.703900    | 4.010590        | 4.949239        | 7.110975        | 6.919039    | 86.441000 | 107.564000 |

## List\_length\_dc\_example\_net'queue\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 8.400000    | 0.828801 | 1.022775 | 1.469505 | 1.429841 | 6        | 10       |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 0.464261    | 0.031128 | 0.038413 | 0.055192 | 0.053702 | 0.391320 | 0.559613 |

## Marking\_size\_example\_net'busy\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 1.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 1        | 1        |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 0.484061    | 0.013468 | 0.016620 | 0.023880 | 0.023235 | 0.445626 | 0.519824 |

## Marking\_size\_example\_net'free\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 1.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 1        | 1        |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 0.515939    | 0.013468 | 0.016620 | 0.023880 | 0.023235 | 0.480176 | 0.554374 |

## CPN Tools Performance Report

Net: D:\courses\BIS-2008\CPN\simulation\mm1.cpn

Number of replications: 10

## Statistics

| Name      | Avrg          | 90% Half Length | 95% Half Length | 99% Half Length | StD          | Min        | Max        |
|-----------|---------------|-----------------|-----------------|-----------------|--------------|------------|------------|
| Flow_time |               |                 |                 |                 |              |            |            |
| count_iid | 1000.000000   | 0.000000        | 0.000000        | 0.000000        | 0.000000     | 1000       | 1000       |
| max_iid   | 1845.200000   | 320.566238      | 395.592378      | 568.379854      | 553.038435   | 1211       | 3088       |
| min_iid   | 0.400000      | 0.299328        | 0.369383        | 0.530723        | 0.516398     | 0          | 1          |
| sum_iid   | 424854.700000 | 47563.021217    | 58694.792140    | 84331.597902    | 82055.362490 | 293109     | 540346     |
| avrg_iid  | 424.854700    | 47.563021       | 58.694792       | 84.331598       | 82.055362    | 293.109000 | 540.346000 |

## List\_length\_dc\_example\_net'queue\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 19.800000   | 3.040301 | 3.751860 | 5.390604 | 5.245104 | 15       | 33       |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 3.526256    | 0.491455 | 0.606477 | 0.871374 | 0.847855 | 2.214570 | 4.693034 |

## Marking\_size\_example\_net'busy\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 1.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 1        | 1        |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 0.821571    | 0.014726 | 0.018172 | 0.026109 | 0.025405 | 0.792455 | 0.867871 |

## Marking\_size\_example\_net'free\_1

|           |             |          |          |          |          |          |          |
|-----------|-------------|----------|----------|----------|----------|----------|----------|
| count_iid | 2002.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 2002     | 2002     |
| max_iid   | 1.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 1        | 1        |
| min_iid   | 0.000000    | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0        | 0        |
| avrg_iid  | 0.178429    | 0.014726 | 0.018172 | 0.026109 | 0.025405 | 0.132129 | 0.207545 |

- $\lambda = 1/100$
- $\mu = 1/80$
- $\rho = 0.8$
- $L = 4$
- $S = 400$

CPN Tools Performance Report

Statistics

| Name                               | Avrg           | 90% Half Length | 95% Half Length | 99% Half Length | StD           | Min        | Max         |
|------------------------------------|----------------|-----------------|-----------------|-----------------|---------------|------------|-------------|
| Flow_time                          |                |                 |                 |                 |               |            |             |
| count_iid                          | 1000.000000    | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 1000       | 1000        |
| max_iid                            | 4899.700000    | 1252.193365     | 1545.259897     | 2220.201002     | 2160.274471   | 2216       | 9389        |
| min_iid                            | 3.300000       | 2.186834        | 2.698646        | 3.877365        | 3.772709      | 1          | 13          |
| sum_iid                            | 1709043.500000 | 507550.790075   | 626339.272858   | 899912.748359   | 875622.763150 | 958796     | 3460629     |
| avrg_iid                           | 1709.043500    | 507.550790      | 626.339273      | 899.912748      | 875.622763    | 958.796000 | 3460.629000 |
| List_length_dc_example_net'queue_1 |                |                 |                 |                 |               |            |             |
| count_iid                          | 2002.000000    | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 2002       | 2002        |
| max_iid                            | 46.800000      | 11.543858       | 14.245612       | 20.467834       | 19.915377     | 26         | 92          |
| min_iid                            | 0.000000       | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 0          | 0           |
| avrg_iid                           | 15.676766      | 4.935593        | 6.090732        | 8.751052        | 8.514848      | 8.628088   | 33.015949   |
| Marking_size_example_net'busy_1    |                |                 |                 |                 |               |            |             |
| count_iid                          | 2002.000000    | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 2002       | 2002        |
| max_iid                            | 1.000000       | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 1          | 1           |
| min_iid                            | 0.000000       | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 0          | 0           |
| avrg_iid                           | 0.954594       | 0.014241        | 0.017574        | 0.025250        | 0.024569      | 0.928707   | 0.995219    |
| Marking_size_example_net'free_1    |                |                 |                 |                 |               |            |             |
| count_iid                          | 2002.000000    | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 2002       | 2002        |
| max_iid                            | 1.000000       | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 1          | 1           |
| min_iid                            | 0.000000       | 0.000000        | 0.000000        | 0.000000        | 0.000000      | 0          | 0           |
| avrg_iid                           | 0.045406       | 0.014241        | 0.017574        | 0.025250        | 0.024569      | 0.004781   | 0.071293    |

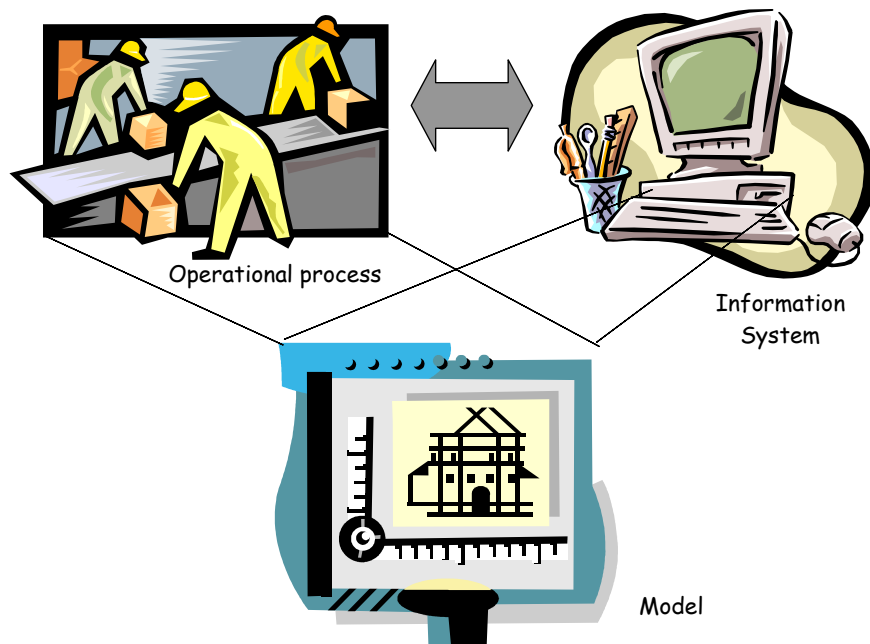
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- $\lambda = 1/100$
- $\mu = 1/99$
- $\rho = 0.99$
- $L = 99$
- $S = 9900$

Note deviations.  
Why?



# Conclusion analysis



- Analysis is typically model-driven to allow e.g. what-if questions.
- Models of both operational processes and/or the information systems can be analyzed.
- Types of analysis:
  - *validation (interactive simulation/gaming)*
  - *verification (state-space analysis, place and transition invariants, siphons, traps, etc.)*
  - *performance analysis (simulation)*